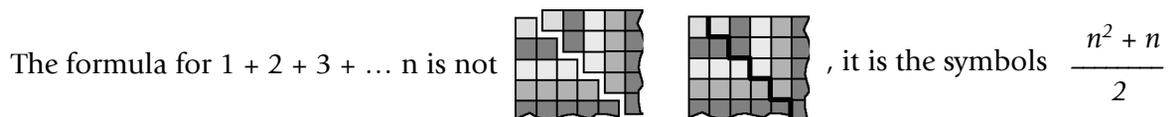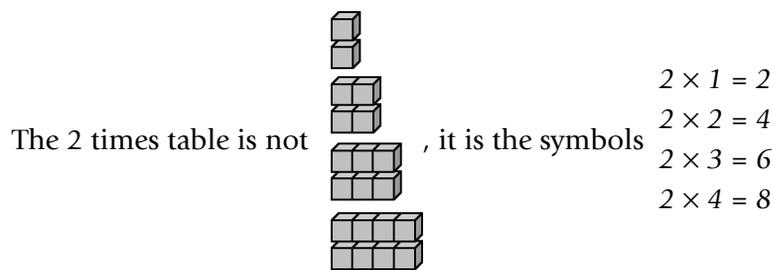# C HAPTER 1

Introduction

- *This book is about teaching mathematics to pupils who have learning differences, not learning difficulties.*

- *Pupils with visual and kinaesthetic learning styles often struggle with a school curriculum that is largely based on print.*

- *The development of 'pictures in the mind' can help all pupils to understand key mathematical concepts.*

a) Different Learning Styles in the Classroom – a Vicious Circle

This is a book about teaching maths to pupils with learning *differences*, not learning *difficulties*. Teaching and learning in our schools is, and always has been, print based. Literacy is all. Other ways of thinking – visual, kinaesthetic, practical – are discounted in the classroom. To become teachers, students must jump over a long series of hurdles, formal and informal, at school, at college and at university. These hurdles consist of print-based activities and assessments that demand a high level of linguistic and symbolic thought but take little account of other ways of thinking and learning. As a result, teachers are rarely selected for their visual or kinaesthetic abilities as these have little impact on their academic achievement. It is their verbal and numerical skills that have opened the doors to success, not their spatial skills. This may make it difficult for teachers to recognise spatial ability in their pupils, so real strengths and aptitudes are neglected as pupils are forced to struggle with a curriculum which is largely presented through printed materials that they find hard to access.

Because the curriculum is so heavily print based, 'proper' school maths is defined as maths that can be printed in a book, and preferably in text. Definitions and proofs that depend on models or dynamic geometry rather than on symbols are second best. So, for example,

The number seven is not or , it is the symbol *7*

The 2 times table is not , it is the symbols

$2 \times 1 = 2$
$2 \times 2 = 4$
$2 \times 3 = 6$
$2 \times 4 = 8$

The formula for $1 + 2 + 3 + \ldots n$ is not , it is the symbols $\dfrac{n^2 + n}{2}$

Pictures and models may be used to support learning, especially in the early stages, but the end point is symbolic. Symbols are easier to print, and they always take precedence over visual or kinaesthetic representations.

But to some children, the numbers and symbols on the page are just squiggles. They can *see* that seven is five plus two, or that twice two two's will fit together to make four two's, or that the sum of the first *n* counting numbers is half the area of a rectangle with sides *n* and *n* + 1. They may not be able to put it into words, but they can see it, and perhaps draw it. It is for these children, and for their teachers, that this book is written.

b) Visual, Auditory, Kinaesthetic

So – what are these different learning styles? There are nearly as many theories about learning as there are researchers writing about it. Steve Chinn offers a useful summary of 'thinking styles in mathematics', and shows how, to some extent at least, the different models overlap and inter-relate (Chinn, 2004, pp59–75). But for general classroom use the VAK model – Visual, Auditory, Kinaesthetic – will serve us well. It is at least as old as Confucius –
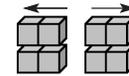
I hear, and I forget;

I see, and I remember;

I do, and I understand.

This model is quite straightforward, and it works well in the classroom so it can provide the theoretical structure we need for the ideas and activities discussed in this book.

The phrase *kinaesthetic learning* is sometimes taken to mean any activity that involves the use of apparatus. This may be considered particularly appropriate for 'slow learners' – at least they will have something to *do* in their mathematics lessons. But if the focus of the teaching is primarily on the correct use of the apparatus, rather than on the mathematical understanding that the apparatus is designed to develop, then it may have only a limited impact. Pupils will just follow the instructions to use the equipment, without necessarily relating what they are doing to mathematics.

Kinaesthetic learning calls for a lot more than a pile of cubes or a pair of scissors and some card. It involves using your whole being, engaging all your senses to feel or imagine what is happening. Visual, aural and kinaesthetic learning are all intertwined: together they can lay down a memory – of movement, feeling, sight and sound – that will be recalled as a total experience, not just as a recited chant. For example, when I think about the number seven I can *feel* the seven in all the fingers of one hand and two fingers of the other. When I factorise, I can imagine pulling apart *eight* to make two sets of *two twos*. And I can feel myself breaking up a 10 by 11 unit rectangle into two halves to find the sum of the first ten counting numbers, 1 + 2 + 3 + … + 10. Because I have done all these activities, and have understood the mathematics that they represent, I do not need to actually hold up my fingers or make blocks of cubes in order to recall them. But what I recall is most certainly not a chant or a formula: it is more like a moving picture – a sort of waking dream. This, I believe, is kinaesthetic learning.

Any teaching idea, no matter how inspirational, can be reduced to 'rote learning' – *I hear and I forget*. On the other hand, the dreariest exercise might be transformed into a basis for real understanding by a teacher who can unpack the underlying concepts and help pupils to understand and use them. We all use a range of learning styles at different times, and the most effective mathematical thinkers are flexible. They try different approaches to the problem in hand, finding out what works best and relating each new idea to what they already know. The *hearing*, the *seeing* and the *doing* support one another, as the pictures, models and activities give meaning to the spoken or written definitions and procedures. Pupils may adopt different styles as they first explore and understand, and then rehearse and apply, each new concept. But for learners who think more easily in pictures and movement than in words and symbols, *seeing* and *doing* may offer access to key mathematical ideas, while too much time spent *hearing* may slam the door shut.

c) Pictures in the Mind

Some people can follow a set of directions easily, but others find it much more helpful to have a visual image. For example, one person might find it easy to follow a written description of a route:

> *Turn left out of the gate, and walk to the T-junction at the end of the road. There you should cross the road and turn right. Take the first left turn, and walk past the school and across the crossroads. You will come to another crossroads, with a church on the corner; there you must turn left. Walk about fifty metres down the road, and the house you want, number 33, is on the right, opposite the post office.*

But another might prefer a map. They find the map easier than the linear series of instructions to understand and to follow, and they can recall it more easily when they need to find their way again along the same route.
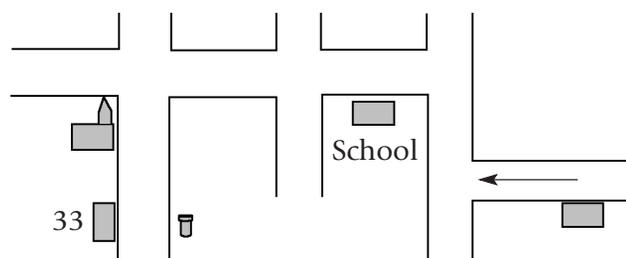
In the mathematics classroom diagrams may be used, but, as we have seen, they are generally subservient to the written, symbolic forms. A map (or its equivalent) is rarely considered to be enough on its own – while a written formula, or a set of rules for carrying out a procedure, can stand alone. Pupils who can take in and remember a series of instructions, or a formula, or the 'rules' for adding fractions or finding the sine of an angle, achieve high grades and feel successful. But those pupils for whom such rules and procedures seem meaningless have great difficulty recalling them, and cannot use them efficiently to solve problems. They may struggle to make sense of the symbols and instructions – or they may just give up in despair. Either way, they do not achieve any real understanding of the concepts that underlie the routines and methods that they are trained to use.

The main purpose of any model or image is to develop the pupils' understanding, so they do not just learn *how* to use a method to solve a problem, but they also understand *why* it works. For example, the image of a number line may help some pupils to see a subtraction as finding the 'distance' between two numbers.

7000 – 4533   **= 2467**

7	60	400	2000
4533 4540	4600	5000	7000
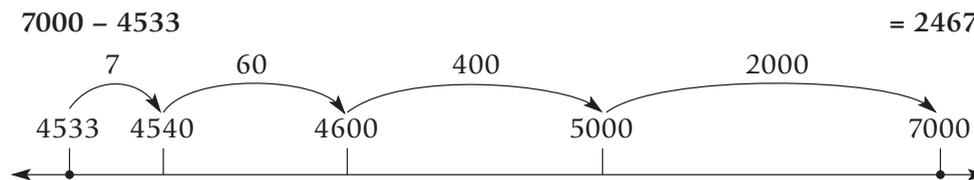
This approach may make much better sense than a standard algorithm –

```
        6 9 9
        ₇̷ ₁0 ₁0 ₁0
7 0 0 0  7̷ 0̷ 0̷ 0̷
4 5 3 3  4 5 3 3
───────  ───────
         2 4 6 7
```

Nought take away three, I can't, borrow one, I can't, borrow one, I can't, borrow one, cross out the seven and put six, make ten in the next column, cross out the ten and make nine, make ten in the next column, cross out the ten and make nine … and so on.

The number line offers far more than this sequential set of 'rules' for getting the right answer. The picture itself – whether printed, drawn, or just imagined – carries within it an explanation of why the method works. In this way, mathematical ideas from the simplest to the most complex can be made manifest, and so become meaningful and memorable to all our pupils – not just to the visual and kinaesthetic learners.

But the number line, like any other model, could be used as just another routine, to be learnt by rote and followed blindly without any understanding of the meaning of each step. Used like this it will be no more helpful, and it will be considerably less tidy, than a numerical algorithm. This book offers a range of models and images that may be useful, particularly for pupils who think more easily in pictures than in words and symbols. By themselves, however, learnt as yet more methods and routines, these models will be useless. If some pupils can, and if they really must, learn and recall mathematics without understanding, then they will do better to acquire the numerical and symbolic routines. These are generally shorter, neater, and easier to memorise and apply than the pictures and models exemplified in this book. For visual and kinaesthetic thinkers, however, this is not an option. They must understand the mathematics that they are taught. Otherwise they may learn … but they will forget.

d) Using Symbols and Understanding Diagrams

Our single most important function as maths teachers is to develop our pupils' understanding of mathematics. Using mathematical language, manipulating numbers and symbols, applying mathematics to solve problems – all this comes into it, of course. But the basis, the rock on which mathematics education is built, is understanding.

Unfortunately, it is terribly easy to teach pupils how to manipulate symbols without understanding. Any teacher with a little determination can teach *how* to add fractions, or *how* to find the area of a circle, or whatever. Pupils can learn to get 'right answers' using symbols and the rules for combining them with little understanding of what they mean. Those who can manipulate symbols quickly and efficiently are often thought to be working at a 'higher level' than those who use diagrams or equipment to work through a problem, making sense of each step on the way. As Keith Devlin puts it, 'Learn how to perform the mumbo-jumbo and you get an A' (Devlin, 2000, p67). A pupil who writes

$$\frac{2}{3} \ + \ \frac{1}{2} \ = \ \frac{2}{3} \times 2 + \ \frac{1}{2} \times 3 = \ \frac{4}{6} \ + \ \frac{3}{6} \ = \ \frac{7}{6} \ = \ 1\frac{1}{6}$$

may be rated much more highly that one who uses a more meaningful graphical approach,
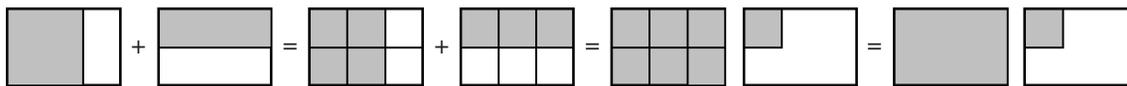
But a pupil who just goes through the steps, and cannot explain *why* $2/3$ is equal to $4/6$, and $4/6 + 3/6$ is equal to $7/6$ which is equal to $1\,1/6$, is not working at a higher level than a pupil who can use, understand and explain the drawings. The diagrams lead, not just to the 'right answer', but to an explanation – a sort of proof that $2/3 + 1/2$ really does equal $1\,1/6$. This involves much more mathematics than any rote learning of meaningless symbolic manipulation. Written numbers and symbols are valuable, and indeed essential, tools for mathematics, but we must always ensure that they are used to express, support and communicate mathematical understanding, not to take its place.

e) Identifying Different Learning Styles

All the pupils in a mathematics classroom – like all the teachers – are able to think visually and kinaesthetically to a greater or lesser degree. There is not a clear-cut divide between spatial thinkers and those who think in words and symbols. The chief difference lies, not in the ability of different pupils to think spatially or numerically, but on the value that is placed on the different thinking styles. But how can teachers spot visual and kinaesthetic ability, and identify pupils who are likely to learn more effectively through models that they can construct and take apart, and through 'pictures in the mind'?

Teachers may well notice the visual and kinaesthetic thinkers in their classroom by their responses to different types of mathematical task. These are the pupils who have found all the nets of a cube before most of the rest of the class have grasped what a net is – but for whom

'seven eights' are 'forty-three' on Tuesday, and 'sixty-two' on Wednesday. With a print-based cur-
riculum they rarely shine – but just occasionally they take everyone (including, quite possibly,
themselves) by surprise with their ability to just see the solution to a problem with which other
pupils are struggling.

There are more formal approaches to the identification of pupils with high spatial ability.
Many schools in the UK routinely screen pupils with the NFER-Nelson (2001) *Cognitive Ability
Tests*. These give three different scores for each pupil: a Verbal Reasoning score, a Quantitative
Reasoning score, and a Non-verbal Reasoning score (Strand, 2003, p5). A pupil with high spa-
tial but low symbolic and numerical ability will be likely to have a high Non-verbal Reasoning
but a lower Verbal Reasoning score. As Steve Strand explains in his book, *Getting the Best from
CAT*, such pupils may have difficulty accessing much of the school curriculum. On the other
hand, pupils with high Verbal Reasoning scores

> tend to have higher national test and examination attainment than pupils with a similar mean
> CAT *score who have their strength on the Quantitative or Non-verbal batteries.*
> *(Strand, 2003, p41)*

He argues that

> *Verbal ability is so crucial to academic success that interventions to directly address … verbal
> weaknesses may be necessary, especially where verbal scores are low.*
> *(Strand, 2003, p49)*

On the other hand,

> *a relative verbal strength can compensate for lower scores in the quantitative and non-
> verbal areas.*
> *(Strand, 2003, p42)*

This evidence again indicates the importance given to verbal ability in our educational system.
High verbal ability can compensate for a lack of other sorts of learning ability – but other
strengths, such as high spatial ability, cannot. Spatial ability is undervalued, and is not usually
exploited to compensate for a lack of verbal ability in enabling pupils to access the curriculum.

Another more specialised series of Spatial Reasoning Tests is also available (Smith and
Lord, 2002). These give teachers the means to routinely identify pupils with strong spatial abil-
ity, and so, with time, may encourage the development of a range of approaches which build
more effectively on their strengths.

f) Assessment for Learning

Assessment drives the curriculum. This is regrettable, certainly. It would be much better if a
broad and balanced curriculum could be established and taught, with assessment following,
not leading, the whole process. But the reality is otherwise. If a topic or a mathematical idea is
never assessed, then in many cases at least it will not be learnt.

Formal written maths tests tend to militate against teaching for understanding, because it is so hard to write a markable test question that actually does assess the *why* rather than the *how* (Clausen-May, 2001, p8). As Black and Wiliam argue in their booklet, *Inside the Black Box*,

> *short external tests … can dominate teachers' work, and insofar as they encourage drilling to produce right answers to short out-of-context questions, this dominance can draw teachers away from the paths to effective formative work.*
> *(Black and Wiliam, 1998, p17)*

'Effective formative work' focuses on pupils' understanding – on finding what they understand now, and building on this to develop their understanding in the future. It is best done informally, in the everyday interchange between teachers and pupils. And since formative assessment focuses on the pupils' understanding, on the *why* rather than the *how*, it supports the use of a full range of teaching and learning styles.

Assessment for learning enables teachers to relate what pupils are learning now to what they have learnt in the past, and to pave the way for what they will learn in the future. The aim is to help pupils at all stages of mathematical development to recognise the links between the different aspects of mathematics and the various individual topics that they meet. An often quoted example is in the connections between decimals, fractions and percentages (Askew et al., 1997, p 26). Pupils learn to 'convert' from one to another, and hopefully to understand the relationships between them.

For visual and kinaesthetic thinkers, however, mathematics is shot through with connections. Multiplication may be seen and understood as area. Fractions, fractions of a turn, angle and telling the time may all be tied together through the image of a clock face. Ratio and place value may be thought of as concepts relating primarily to mathematical similarity – to shapes and solids that expand and contract without distorting. And so on. So for a visual and kinaesthetic thinker the distinctions between Number, Algebra, Shape, Space and Measures, and Data Handling may be very blurred. But are these, in any case, strictly mathematical distinctions? They are useful administrative and organisational categories, and they lie at the heart of the school mathematics curriculum. But they do not, perhaps, lie at the heart of mathematics.

This book is written to help teachers to recognise those pupils who think more easily in pictures and movement than in words and symbols, and to help them to find or build the visual and kinaesthetic 'pictures in the mind' that they need. There is not just one model that will work for every topic for every pupil – there are many possibilities. The chapters that follow offer a range of suggestions, relating to a variety of topics at different levels, but teachers may well have others that work better in their classrooms. The ideas put forward here are intended primarily as illustrations of an approach – an approach that seeks out 'models to think with' that can help pupils to develop their understanding. Some of these ideas may be useful for particular pupils, but they are only a start. Teachers – and the pupils themselves – need to be constantly alert, on the lookout for images and models that will represent and explicate specific concepts. You can start with practically any resource or activity, and see how it could be adapted for visual and kinaesthetic learners. It is the approach that matters, not the details of particular activities or materials. Making mathematical concepts manifest with pictures and models will help all pupils – even those who could, if it were really demanded of them, learn and remember routines for getting 'right answers'.

Introduction – Key Points

■ Children have different learning styles – Visual, Auditory and Kinaesthetic (VAK).

■ The school curriculum is heavily print based. This favours auditory learners.

■ Visual and kinaesthetic thinking and learning styles are under-valued in the classroom.

■ Visual and kinaesthetic learners need a 'picture in the mind' to hang their thinking on.

■ A visual approach is worthwhile only if it is based on understanding.

■ Mathematical symbols are there to express, support and communicate understanding, not to take its place.

■ Teachers can identify pupils with different learning styles informally, through observation, or more formally, using a range of tests.

■ Assessment for learning supports teaching for understanding.

■ Appropriate models will help pupils to recognise links between different aspects of mathematics.