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Managing Your Classroom

STRATEGY 1: Create your own support network as soon as you begin your first teaching job.

What the Research Says

Teacher retention is a major concern of school districts nationwide. Inexperienced teachers are leaving the profession at an alarming rate, and they are being replaced by even less experienced teachers. This is creating a dangerous cycle that can have a profound effect on the learning community. Providing support for new teachers must be a priority of every district and school administration.

The Miami-Dade Public Schools have kept detailed records of school personnel and have conducted research on teacher retention. While they have found that turnover has significantly increased over the past decade, they did not attribute it to the salary scale. In fact, the research suggested that turnover was linked to "deterioration in the work environment with across-the-board policy changes that affect teachers."[1] It is generally accepted that inexperienced teachers are overwhelmed in their first year on the job. Though many contributing factors are cited, it is clear that lack of support is at the top of the list. The small school model typically has only a few math teachers, on staff. It is very difficult to obtain a balanced perspective on mathematics instruction from such a small group. The larger schools have many more teachers, but oftentimes are overcrowded. Teachers do not have common work areas, and there is not a feeling of community or a healthy collegial atmosphere. New teachers are left to fend for themselves. Simple tasks like making copies, obtaining textbooks, gaining access to technology, and obtaining supplies seem insurmountable. It is this culture and atmosphere that drives dedicated, well-intentioned individuals from the profession.

Teaching to the NCTM Standards

The NCTM Professional Standards addresses the development of the individual as a teacher of mathematics. Teachers should possess

> the growing sense of self as a teacher, and the continual inquisitiveness about new and better ways to teach and learn that serve teachers in their quest to understand and change the practice of teaching . . . they need to become the teacher of students under the guidance and support of both a cooperating practitioner and a mathematics educator.[2]

Classroom Applications

When you arrive at a school, your first goal should be to find a collaborative buddy—an experienced teacher who can be relied on to help you with some of the mundane aspects of your job. We have found that this is the single most important strategy to help new teachers survive their first year on the job. Newly licensed teachers are amazed at the hurdles they must face in their first year. Oftentimes, the training that they received in education programs did not adequately prepare them for the situation in which they now find themselves. Thus, having a buddy is essential. We suggest that new teachers use their buddies in many ways in addition to just procuring supplies. Collaborative buddies can share some aspects of the culture of the school. New teachers may wish to ask their buddies for some classroom management strategies, including how they take attendance, how they assign seats, how they assign homework, how they check homework, how often they give tests, what the format of their tests is, what additional resources are available for the students and teachers, whether there are class sets of calculators available, and so on. The best resource new teachers can have is the collective genius and experience of their colleagues.

Precautions and Possible Pitfalls

Remember that you have your own style of teaching and communicating. Although it is wonderful to solicit advice from veteran teachers, keep in mind that you have to adapt their suggestions to work with your particular strengths and weaknesses. Moreover, it is advisable to have many collaborative partners instead of just one. Then you can balance the ideas and suggestions and choose those that fit your personality and style.

Source

Burke, D., Cavalluzzo, L., Hansen, M., Harris, J., Lien, D., & Wenger, J. (2004, March). *Relative pay and teacher retention in Miami-Dade County public schools: Summary and research* (Report). Miami: Education Center Institute for Public Research.

STRATEGY 2: Before beginning a lesson, put an outline of what you are going to cover on the blackboard.

What the Research Says

Learning is more meaningful when students know in advance what is going to be covered in a lesson and how the teacher organizes the information to be learned. Seeing an outline on the board stimulates students' thinking about the various topics and helps them activate their prior knowledge about the topic. The connection between existing and new knowledge is an essential component of meaningful learning. The suspense or curiosity factor about the unknown is a useful device for motivation, and should be balanced with the prescription for the lesson.

Teaching to the NCTM Standards

The NCTM Learning Principle requires that students must "actively build new knowledge from experience and prior knowledge."[3] The research suggests that by providing an outline of what will be covered in class, students can begin to think about existing knowledge and concepts that are related to the day's lesson. For quite some time it has been standard practice to provide (or elicit) the "AIM" of the lesson and write it clearly on the board so that students will know the exact goal of the lesson. Providing an outline, as indicated above, takes that to a deeper level, providing an in-depth inventory of the concepts and skills that students will be learning. This can also be helpful to both the student and teacher to assess whether the stated goals of the lesson have been met.

Classroom Applications

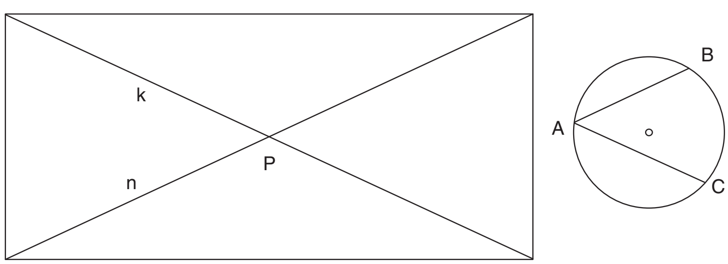
There is a very clever method of teaching the theorems in geometry that deals with the measurement of an angle related to a circle. As a first step you might do a class on the various types of angles. Begin the lesson by putting an overview (advance organizer outline) of what you will be covering on the chalkboard. Example:

> Theorems in geometry involving various types of angles related to a circle:
>
> a. an inscribed angle
> b. an angle formed by two chords intersecting in the circle (not at the center)
> c. an angle formed by two secants intersecting outside the circle
> d. an angle formed by two tangents intersecting outside the circle
> e. an angle formed by a tangent and a secant intersecting outside the circle
> f. an angle formed by a chord and a tangent meeting at the point of tangency

This advance organizer outline will stimulate students to think about what they already know about angles (e.g., what types of angles they know of, such as a triangle; have they ever heard of an inscribed angle; are they familiar with the concepts of tangents, secants, etc.). It will also show students how you organize the ideas you are discussing—where one topic stops and another begins. You may want to include illustrations of each type of angle on the chalkboard so students can visualize the concepts and see whether or not they recognize them.

This activity is designed to demonstrate all of the measurements of the variations of an angle related to a circle. It can be carried out very nicely by cutting a circle out of a piece of cardboard and drawing a convenient inscribed angle on it. The measure of that angle should be the same as that formed by two pieces of string that are affixed to a rectangular piece of cardboard (see Figure 1.1).

Figure 1.1



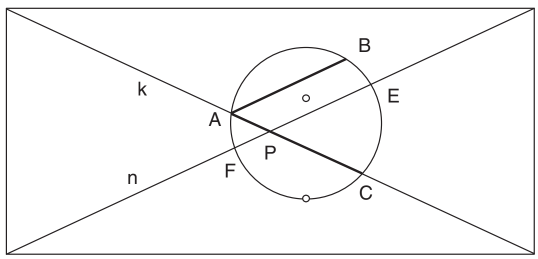
It is assumed that the theorem establishes that the measure of an inscribed angle of a circle is one-half the measure of the intercepted arc.

By moving the circle to various positions we will be able to find the measure of an angle formed by

- two chords intersecting inside the circle (but not at its center)
- two secants intersecting outside the circle
- two tangents intersecting outside the circle
- a secant and a tangent intersecting outside the circle
- a chord and a tangent intersecting on the circle

We begin with demonstrating the relationship between the arcs of the circle and the angle formed *by two chords intersecting inside the circle* (but not at its center). Position the cardboard circle so that A and C are on *k*, as in Figure 1.2.

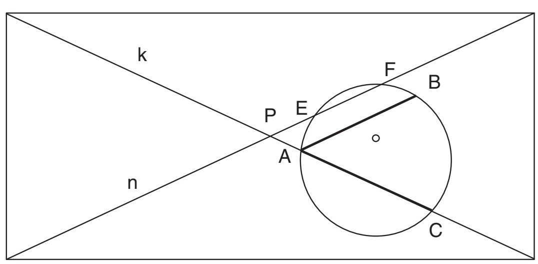
Figure 1.2



Notice that $m\angle A = \frac{1}{2}\widehat{BEC}$, and $m\angle A = m\angle EPC$. Therefore $m\angle P = \frac{1}{2}m\widehat{BEC} = \frac{1}{2}(m\widehat{BE} + m\widehat{EC})$. But, because parallel lines cut off congruent arcs on a given circle, $m\widehat{BE} = m\widehat{AF}$. It then follows that $m\angle p = \frac{1}{2}(m\widehat{AF} + m\widehat{EC})$, which shows the relationship of the angle formed by two chords, $\angle P$, and its intercepted arcs, $\widehat{AF}$ and $\widehat{EC}$.

Consider next the angle formed by *two secants intersecting outside the circle.* Place the cardboard circle into the position shown in Figure 1.3.

Figure 1.3

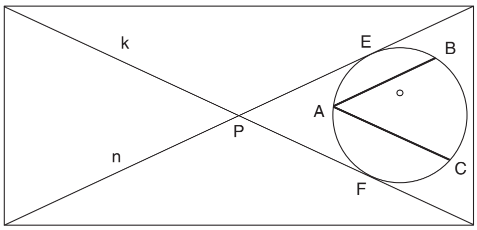


Begin by remembering that $m\angle P = \frac{1}{2} m\widehat{BC}$ and $m\angle FPC = m\angle A$. Because $m\widehat{AE} = m\widehat{BF}$, we can add it to and subtract it from the same quantity without changing the value of the original quantity. Thus,

$$m\angle P = \frac{1}{2}(m\widehat{BC} + m\widehat{BF} - m\widehat{AE} = \frac{1}{2}(m\widehat{FBC} + m\widehat{AE}).$$

In a similar way we can demonstrate the relationship between an angle formed by *two tangents intersecting outside the circle* and its intercepted arcs. We move the cardboard circle into the position shown in Figure 1.4.

Figure 1.4



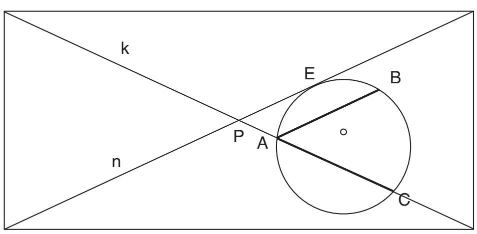
In this case, the equality of arcs, $\widehat{AE}$ and $\widehat{BE}$ as well as that of arcs $\widehat{AF}$ and $\widehat{CF}$ is key to demonstrating the desired relationship.

We have

$$m\angle P = m\angle A = \frac{1}{2} m\widehat{BC} = \frac{1}{2}(m\widehat{BE} + m\widehat{BC} + m\widehat{CF} - m\widehat{AE} - m\widehat{AF})$$
$$= \frac{1}{2}(m\widehat{EBCF} + m\widehat{EAF}).$$

Again, by sliding the cardboard circle to the following position (see Figure 1.5) we can find the measure of the angle formed *by a tangent and a secant intersecting outside the circle.*

Figure 1.5

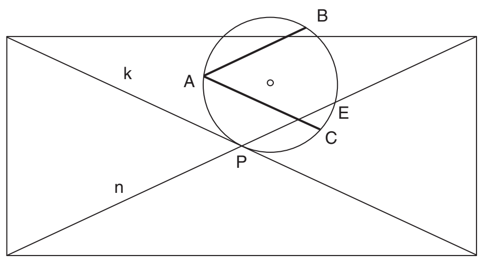


This time we rely on the equality of arcs. We get the following by adding and subtracting these equal arcs:

$$m\angle P = m\angle A = \frac{1}{2}m\widehat{BC} = \frac{1}{2}(mBC + m\widehat{BE} - m\widehat{AE}) = \frac{1}{2}(m\widehat{EBC} - m\widehat{AE}).$$

To complete the various possibilities of positions for the cardboard circle, place it so that we can find the relationship between *an angle formed by a chord and a tangent intersecting at the point of tangency* and its intercepted arc (see Figure 1.6).

Figure 1.6



The crucial arc equality this time is $mAP = CP$, and $mAP = BE$. We begin as before:

$$\begin{aligned} m\angle P = m\angle A = \frac{1}{2}m\widehat{BEC} &= \frac{1}{2}(m\widehat{BE} + m\widehat{EC} + m\widehat{PC} - m\widehat{AP}) \\ &= \frac{1}{2}(m\widehat{EC} + m\widehat{PC}) = \frac{1}{2}m\widehat{PCE}. \end{aligned}$$

This activity can also be done quite nicely with a computer drawing program such as *Geometer's Sketchpad.*

Precautions and Possible Pitfalls

As mentioned above, this procedure for beginning a lesson has its advantages, but also the disadvantage of removing the "controlled surprise" factor from the lesson. It is important for the teacher to weigh this disadvantage against the gains when using this approach. Such professional judgments are always necessary when planning a lesson, but particularly in this case.

Source

Ausubel, D. (1960). The use of advance organizers in the learning and retention of meaningful verbal learning. *Journal of Educational Psychology, 51*, 267–272.

STRATEGY 3: Make realistic time estimates when planning your lessons.

What the Research Says

Teachers need to have excellent time management skills for students to learn effectively. It is sometimes said that "time + energy = learning." Sometimes teachers confuse time allocated for instruction with time students are actually engaged in learning. The concept of engaged time is often referred to as "time-on-task." Teachers often fail to take into account the time they end up devoting to managing student behavior and managing classroom activities. Teachers need to take this distinction between allocated and engaged time into account when estimating how much time it will take for students to learn a particular set of material. It's the time students actually spend learning that is the key to the amount of achievement.

Teaching to the NCTM Standards

In the NCTM *Research Companion to Principles and Standards for School Mathematics,* the subject of using time effectively in the classroom is addressed to ensure that lessons are planned to allow for students both to "learn to communicate mathematically and to communicate to learn mathematics. This requirement calls for planning that involves capitalizing on what students do and directing their activities toward important mathematics issues."[4] Effective planning is not simply choosing the right number of problems to support the lesson, but as the application below suggests, making important decisions about what to include in a given lesson so that meaningful learning takes place. "At the beginning of a discussion, the teacher might call on specific students selected in advance because he or she anticipates that a comparison of their solutions might lead to substantive mathematical conversation that advances the pedagogical agenda."[5] Allowing adequate time for student communication and involvement is key to planning a successful lesson.

Classroom Applications

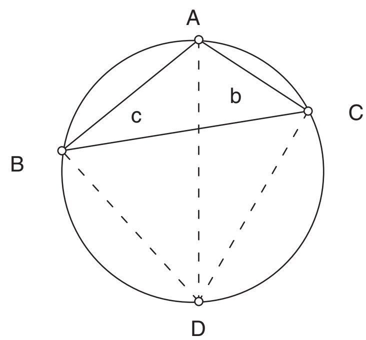
Suppose you are planning a lesson on the introduction to the Law of Sines. You would like to develop or derive the Law, and you would like to have ample time to apply the Law to "practical" examples as well as the drill that typically follows the introduction of the Law. To fit this into a normal 50-minute lesson you might either relegate a more serious inspection of the derivation to a homework assignment and simply introduce the Law of Sines and its applications to the triangle, or you might search for a concise derivation of the Law, of Sines such as the following:

Consider the circumcircle of ΔABC with diameter $\overline{AD}$.

$$\text{Diameter AD} = \frac{AC}{\sin\angle ADC} = \frac{AC}{\sin\angle ABC}$$

$$\text{Diameter AD} = \frac{AB}{\sin\angle ADB} = \frac{AB}{\sin\angle ACB}$$

Figure 1.7



Therefore, $\frac{AB}{\sin\angle ADB} = \frac{AB}{\sin\angle ACB}$, which can then by followed in a similar way to the third part of the Law of Sines. This very concise proof will allow the teacher ample time to do a complete lesson on a topic which would otherwise require more than one lesson to introduce. In other words, it is wise for a teacher in planning a lesson and trying to ensure that there is enough time to do the lesson to also search for alternative methods which might be more concise and allow for a more streamlined lesson.

Precautions and Possible Pitfalls

Be sure to plan time in your "clever" lesson time for your students to digest the cleverness of proofs or demonstrations which may not be in the textbook. This will ensure that they have time to take complete notes of the development you are doing in the lesson so that review at home is then possible.

Source

Brophy, J. (1988). Research linking teacher behavior to student achievement: Potential implications for instruction of chapter 1 students. *Educational Psychologist, 23*, 235-286.

STRATEGY 4: Make classroom activities flow smoothly.

What the Research Says

Kounin's classic study of classroom management compared effective with ineffective teachers. The effective teachers' classes did not have many problems, while continuous disruption and chaos characterized the ineffective teachers' classes. By observing effective and ineffective teachers' classes, Kounin discovered that the major difference between them was in preventing problems rather than handling problems once they arose. One way teachers prevented problems was by making sure students were not sitting around waiting for the next activity, but were engaged in meaningful work all the time. Effective teachers had smooth transitions between activities, they conducted activities at a flexible and reasonable pace, and their lessons involved a variety of activities.

Teaching to the NCTM Standards

The NCTM Professional Standards addresses the effective learning environment for mathematics instruction. The standard includes "providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems; using the physical space and materials in ways that facilitate students' learning of mathematics."[6] Time is the most precious resource a teacher has in delivering mathematics instruction. Often the difference between a satisfactory lesson and an outstanding lesson is the one or two "extra" applications that the students can explore during a lesson. These applications are typically more challenging and their inclusion is possible because of outstanding time management by the instructor. The application below gives one method of maximizing the use of time to review the homework assignment from the previous day's lesson. A skillfully prepared lesson should be artfully choreographed to provide a seamless transition from one activity to the next.

Classroom Applicationss

The smooth beginning of a class is quite obviously an important aspect of an effective lesson. Rather than spending time on having students copy their homework on the chalkboard at the beginning of a lesson, a smooth beginning would have them immediately bring their work to the class by showing it on an overhead projector (or video monitor). Students should have their model solutions on a transparency when they enter the classroom (done the evening before as part of their homework) so that they can present their work without taking class time to copy it onto a transparency.

Another example of a smooth transition is for the teacher to structure the order of topics to be presented in a logical way so that one comfortably leads into the next, or so that one situation is an elaboration of its predecessor, and so on. Teachers should make sure transitions are smooth both in terms of topics presented (content) and student activities (process). With the teacher conscious of these transitions, the likelihood of them happening is greatly increased.

Precautions and Possible Pitfalls

Be aware of the smoothness of a lesson's start. Student behavior and overall attitude will determine the appropriateness of a lesson's level and tone. Be prepared to make modifications and adjustments to the students' style (within reason, of course), and remember, there is not one best method. The "best method" may vary from class to class!

Some general suggestions follow:

1. Avoid interrupting students while they are busy working.
2. Avoid returning to activities that have been treated as finished.
3. Avoid starting a new activity before finishing a preceding activity.
4. Avoid taking too much time when beginning a new activity. Slowing down can breed trouble.

Source

Kounin, J. (1970). *Discipline and group management in classrooms.* New York: Holt, Rinehart & Winston.

STRATEGY 5: Have "eyes in the back of your head" so you notice misbehavior at an early stage.

What the Research Says

In Kounin's classic study of classroom management, the major difference between effective and ineffective teachers' classrooms was in preventing problems rather than handling problems after they arose, thus causing continuous disruption and chaos. By observing effective and ineffective teachers' classes, Kounin discovered that one way teachers prevented problems was by "with-it-ness." This was the teachers' ability to be aware of what individual students were doing at all times. They were not missing what was going on around the classroom while they were working with a few individual students. These teachers were described as having "eyes in the back of their heads." With this alertness, teachers were able to prevent minor problems from becoming major problems and, when a problem did arise, they were able to pinpoint the cause or culprit and put a quick end to the disturbance. This enabled the teacher to deal expeditiously with the problem at hand and effectively avoid its recurrence.

Teaching to the NCTM Standards

The NCTM Professional Standards for Teaching Mathematics state that teachers should "monitor students' participation in discussions and deciding when and how to encourage each student to participate."[7] The teacher's role in monitoring the class is essential to ensure that all members of the class are actively engaged in the lesson. Classroom discussion should not be exclusively limited to the best students in the class. The NCTM Professional Standards for teaching state:

> It is a key function of the teacher to develop and nurture students' abilities to learn with and from others. . . . Classroom structures that can encourage and support this collaboration are varied: students may at times work independently, conferring with others as necessary; at other times students may work in pairs or small groups.[8]

The applications and suggestions below provide a variety of "tricks," techniques, and methods by which the teacher can keep the entire class on task during a lesson. Engaging all of the members of the class is a proactive measure that supports teaching and learning and prevents students from misbehaving in class.

Classroom Applications

There are numerous "tricks" that teachers can use to move them in the right direction with respect to this sort of with-it-ness:

- Remember the spot from which you address the class is the temporary "front of the room." Constantly move around the classroom;

don't stand in one spot. This allows most of the class to be at the "front of the room" at least for a part of the lesson.

- Talk to the class even as you write. This will remind them that your thoughts are still on them and not on the board-work. Don't turn your back to the class, except for very brief intervals.
- Don't talk to one student while ignoring the rest of the class. If you speak to one student, do so while looking at the rest of the class at the same time.
- Don't always call on volunteer responses to teacher questions. Call on those students who tend to look away from the teacher as soon as a question is asked.
- Frequently make eye contact with students who tend to cause disruptions so they know you're paying attention to them.
- While working with one cooperative learning group, look around the classroom to see how other groups are working. Make comments as necessary. Let them know that although you are working with one group, you are not ignoring what the other groups are doing.

Precautions and Possible Pitfalls

The way to be "with it" is an individual phenomenon. A teacher must do that which is consistent with his or her personality. To do otherwise would be counterproductive because students are very adept at picking up unnatural behavior. Bear in mind that there are no perfect universal solutions to maintaining proper class control. The above are merely suggestions that ought to be modified to meet individual personalities and strengths.

Source

Kounin, J. (1970). *Discipline and group management in classrooms.* New York: Holt, Rinehart & Winston.

STRATEGY 6: Help students develop self-control to enhance their thinking and independence, as well as to ease your own workload.

What the Research Says

Just a few small changes in your methodology could provide an increase in students' self-control. The increase of students' self-control does not release the teacher from his or her outside control. Outside control by the teacher is the basic prerequisite for step-by-step cultivation of self-control. Gradually transfer your control and guidance to students as they develop their own control and feelings of responsibility. In a study about students' self-control, students reported that their lack of self-control makes them feel uncertain about whether they really reached an educational objective. That's why students want external control by the teacher, and that's precisely why teachers need to guide students in realizing self-control.

The research has shown that:

1. High-achieving students have better self-control than students who have learning weaknesses. However, good students sometimes think that they do not need to practice self-control.
2. Preplanned self-control is hard to observe in students. When self-control is observed, it tends to be more reactive than proactive.
3. The more students are proactive in their self-control, the better they are in reacting with self-control.
4. Girls show a stronger tendency toward self-control than boys. Boys tend to skip steps of self-control or do it superficially.
5. Within situations of high demands (such as tests), students realize a greater degree of self-control. Self-control during homework tends to be considered superfluous.
6. When teachers control students' behavior, students tend to adapt to it and refrain from self-control.
7. Teachers' efforts to encourage students' self-control focus only on reactive or result-related self-control.
8. Students know only about techniques of result-related or reactive self-control. These include self-checking, using reference books, using calculators, or verifying the results of a calculation. Low-achieving students tended to mention verifying a calculation as the technique of self-control. Only a few high-achieving students identified making rough estimates as a technique of proactive self-control.

Teaching to the NCTM Standards

The NCTM Principles and Standards for School Mathematics state that, "A major goal of school mathematics programs is to create autonomous learners."[9] Giving students the responsibility for their own self-control is an important step toward allowing students to grow into independent learners. "Students learn more and learn better when they can take control of their learning by defining their goals and monitoring their progress."[10] "Effective learners recognize the importance of reflecting on their thinking and learning from their mistakes."[11] All of the applications below support this theme and advance the goal of creating autonomous learners.

Classroom Applications

Practice continuous methods of self-control like:

- Making rough estimates (Do not trust blindly in the calculator!)
- Using mathematical theorems
- Procedures for drawing representations
- Procedures for graphic representations
- Using templates and measuring instruments

Practical ways for step-by-step improvements in students' self-control consist of the following:

- Students make mutual comparisons of their answers and solutions strategies.
- When one student presents his or her way of solving a problem, another student should give feedback to the problem solver.
- Combine assignments with elements of playful self-control. This is suitable particularly for students with learning weaknesses or students with impulsive work habits.

At the end of this strategy you will find an example of an exercise with such a combination between assignment and playful self-control.

- Give assignments that force students to engage in proactive self-control:
  - Design a task that contains superfluous information.
  - Assign a problem or task that is not solvable or is solvable only under certain conditions.

With each of these techniques of self-control, a teacher is likely to complain about loss of time. However, the students' developing self-control abilities will save time in the long run.

Precautions and Possible Pitfalls

Strict demands on students to use techniques of self-control incessantly or indiscriminately can backfire. Students who already understand an algorithmic procedure will view the demanded checking as only a mechanical (and therefore meaningless) activity. As a result of this, these students may devalue the self-control.

Source

Frank Fischer & Kittlaus Bernd. Ergebnisse von Untersuchungen zur Selbstkontrolle bei Schülern der 7. Klasse im Mathematikunterricht (Results of investigations about self-control of 7th grade students in math classes). *Mathematik in der Schule,* 29 Jg., Heft 11 (1991) S. 761–768.

STRATEGY 7: Do more than one thing at a time.

What the Research Says

Kounin's classic study of classroom management compared effective with ineffective teachers. The effective teachers' classes did not have many problems while the ineffective teachers' classes were characterized by continuous disruption and chaos. By observing effective and ineffective teachers' classes, Kounin discovered that the major difference between them was in preventing problems rather than handling problems once they arose. One way teachers prevented problems was by "overlapping" activities, or supervising and keeping track of several activities at a time. In order to successfully overlap activities, effective teachers continuously monitored what was going on in the classroom.

Teaching to the NCTM Standards

The NCTM Professional Standards for Teaching and Learning ask teachers to reflect upon the following question: "How well are the tasks, discourse, and environment working to foster the development of students' mathematical literacy and power?"[12] The effective mathematics teacher recognizes the varying ability levels of the students in the class and must adapt to provide a differentiated instructional model to challenge and engage all of the members of the class and keep them on task for the duration of the period. "Teachers must monitor classroom life using a variety of strategies and focusing on a broad array of dimensions of mathematical competence."[13]

Classroom Applications

There is an adage in the teaching world that the teacher needs to move continuously about the classroom and be omnipresent. A critical time for teachers to make their presence known is at the beginning of a lesson. While students are putting homework problems on an overhead transparency or on the blackboard, the teacher is free to work with individual students. While a teacher or student is collecting homework assignments, the teacher can be introducing the class to the next topic by posing a problem or question to tap and review students' prior knowledge of the topic they will discuss next. While walking around the classroom discussing a topic, a teacher can glance at students' desks to check for homework or to make sure students are looking at the appropriate material. Teachers can also discuss how to solve a problem while walking around and showing their presence to make sure students do not misbehave.

Precautions and Possible Pitfalls

The key thing to bear in mind is not to spread yourself too thin when assuming more than one responsibility at a time. In addition, don't move around the classroom so much that the movement becomes a distraction for students.

Source

Kounin, J. (1970). *Discipline and group management in classrooms.* New York: Holt, Rinehart & Winston.

STRATEGY 8: Work directly with individual students as often as possible.

What the Research Says

Frequent contact between teachers and students helps students develop academically and intellectually. Rich teacher-student interaction creates a stimulating environment, encourages students to explore ideas and approaches, and allows teachers to guide or mentor individual students according to their individual needs.

Teaching to the NCTM Standards

The *NCTM Handbook of Research on Mathematics Teaching and Learning* states that students "construct their own mathematical knowledge rather than receiving it in finished form from the teacher or the textbook."[14] Thus, what students are "thinking" about mathematics is what they understand about it through "their own internal representation of the interactions with the world and build their own networks of representations."[15] It is very difficult for teachers to assess this understanding without spending some time interacting directly with students. The normal classroom discourse typically does not allow for this depth of analysis by the teacher. When the teacher works directly with the student, there is a noteworthy efficiency in teacher assessment of the individual student and an outstanding opportunity to provide clarification and support where needed. This enables students to have a continuity of understanding within their existing network of mathematical knowledge. A direct result of this dynamic is that less has to be remembered. "If something is understood, it is represented in a way that connects it to a network. The more structured the network, the fewer individual pieces need to be retrieved separately."[16]

Classroom Applications

Working with individual students in a traditional classroom setting is not practical for long periods of time. While students are working individually on an exercise, the teacher should visit with individual students and offer them some meaningful suggestions. Such suggestions might include hints on moving a student who appears frustrated or bogged down on a point toward a solution. These private comments to students might also be in the form of advice regarding the form of the student's work. That is, some students are "their own worst enemy" when they are doing a geometry problem and working with a diagram that is either so small they cannot do anything worthwhile with it, or is so inaccurately drawn that it proves to be relatively useless. Such small support offerings will move students along and give them that very important feeling of teacher interest.

In some cases, when a student experiences more severe problems, the teacher might be wise to work with the individual student after classroom hours. In the latter situation, it would be advisable to have the student describe his work as it is being done, trying to justify his procedure and explain concepts. During such one-on-one tutoring sessions, the teacher can get a good insight into the student's problems. Are they conceptual? Has the student missed understanding an algorithm? Does he have perceptual difficulties? Spatial difficulties? And so on?

Precautions and Possible Pitfalls

To work with individual students and merely make perfunctory comments when more might be expected, could be useless when considering that the severity of a possible problem might warrant more attention. Teachers should make every effort to give proper attention to students when attempting to react to this teaching strategy. They should keep the student's level in mind so that where appropriate, teachers can add some spice to the individual sessions by providing carefully selected challenges to the student so that there may be a further individualization in the learning process. Make sure good students don't get bored. Challenge them by giving them more difficult problems to solve, having them tutor other students, or having them evaluate alternative approaches to solving a problem.

Source

Pressley, M., & McCormick, C. (1995). *Advanced educational psychology.* New York: HarperCollins.

STRATEGY 9: Use classwide peer tutoring to help your students learn, whether or not they have learning disabilities.

What the Research Says

A whole classroom of students helping other students has been found to be an efficient and effective method of enhancing achievement. Twenty teachers participated in a study of classwide peer tutoring with forty classrooms in elementary and middle schools. Half of the schools implemented classwide peer tutoring programs and half did not. Both urban and suburban schools participated in the study. Students came from diverse backgrounds, both culturally and linguistically. There were three different categories of students: average achievers, low achievers without learning disabilities, and low achievers with learning disabilities.

The peer tutoring programs were conducted three days a week, for thirty-five minutes a day, for fifteen weeks. Stronger students were paired with weaker students. Teachers reviewed each pair to ensure they were socially compatible. In all pairs, students took turns serving in the roles of tutor and tutee. Student pairs worked together for four weeks; then teachers arranged new pairings. Teachers received training on how to train their students to be tutors. Tutor training included teaching students how to correct each other's errors. Achievement tests were administered before and after the peer tutoring program. Regardless of whether students were average achievers or low achievers with or without learning disabilities, students in the peer tutoring classrooms achieved at higher levels than those in the classrooms without classwide peer tutoring.

Teaching to the NCTM Standards

The NCTM Professional Standards encourage and expect that students

> work independently or collaboratively to make sense of mathematics. . . . Students' learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas. It is a key function of the teacher to develop and nurture students' abilities to learn with and from others.[17]

Powerful learning communities employ peer tutoring as an important component of teaching and learning. Empowering students to help one another advances the goal of having students become autonomous learners.

Classroom Applications

There are many areas in mathematics that lend themselves to a peer tutoring program. When there is a skill to be learned and all that one needs is experience with success (i.e., drill with immediate feedback), then peer tutoring could provide an efficient way to monitor and support a student trying to master the skill. Say a student has difficulty with factoring, and part of his problem is recognizing which type of factoring is called for. To compound the problem, more than one type of factoring may be used, making this doubly confusing. Here a peer tutor (under the guidance of a teacher) can be quite beneficial. A student who has difficulty doing geometric-theorem proofs could find that a peer tutor is a genuine asset. In addition, the tutor, in explaining the proof to the student, is also provided with an opportunity to strengthen his or her own understanding of the concept of proof (a higher-order thinking skill) and with the role of proof in mathematics. Thus, there is often mutual benefit in a peer tutoring program.

Precautions and Possible Pitfalls

A tutor training program offered by the teacher must precede peer tutoring. Tutors must be given some instruction on how to conduct the sessions, what sort of difficulties to look for on the part of the tutee, and what points to stress in the sessions (based on the teacher's assessment of the class). Any individual difficulties on the part of the tutees should be mentioned to the tutor prior to the sessions. Tutors should be taught to guide student learning, and *not* merely solve problems for students. Students with severe learning disabilities may be too disruptive to benefit from classwide peer tutoring unless the tutors first receive individualized instruction from learning disabilities specialists.

Source

Fuchs, D., Fuchs, L., Mathes, P. G., & Simmons, D. (1997). Peer-assisted learning strategies: Making classrooms more responsive to diversity. *American Educational Research Journal, 34*(1), 174–206.

STRATEGY 10: Encourage students to be mentally active while reading their textbooks.

What the Research Says

When comparing students who are good at understanding what they read with students who are poor at understanding what they read, research shows that good comprehenders are more mentally active than poor comprehenders. Mental activities that characterize good comprehenders include skimming, self-questioning, rereading, inferring, and visualizing. In addition to using such strategies for actively processing the text, good readers tend to coordinate their reading strategies to achieve comprehension.

Teaching to the NCTM Standards

Teachers of mathematics rely on their textbooks in their day-to-day teaching. Decisions on what to teach and how to teach it are informed by the choice of textbook that students are using. Educators must teach students how to use the textbook effectively. One of the primary functions of the textbook is to provide exercises for students to solve, as well as a variety of examples that highlight methods of solution. The approach that a particular author takes to the teaching and learning of mathematics influences the scope and sequence of the topics. In addition, the manner in which various process strands are incorporated into the content strands further magnifies the importance of choosing an appropriate text and using it correctly. Because the NCTM values both process and content, it is essential that teachers instruct students to use the textbook to compare and contrast solutions to a wide variety of problems. Students should not view the textbook simply as a source of problems, but rather as a source for seeking solutions. Thus, actively engaging a student with a good textbook provides the student with a safety net and affords the student the opportunity to take academic risks in problem solving. Where appropriate, sample exercises should be used to generate discourse on alternative solutions that are available.

Classroom Applications

It is unfortunate that most students do not use their mathematics textbook in the way authors would like them to be used. Typically, students only use textbooks to complete homework assignments or to prepare for a test. This use shortchanges many students, for the textbook could very well (and often does) provide alternative explanations to a concept explained by the teacher in class. Students would be well advised (and should even be urged) to read the explanatory material covered in class, for it is quite conceivable that a student's notes are not always complete or truly reliable. Reading a mathematics book is clearly not like reading a novel. The teacher ought to take time out from the normal mathematics instructional program to focus on the way a mathematics textbook ought to be read. By taking a small snippet of time from each of a series of lessons in order to consider the textbook and how it should be used, the teacher will be making the review (via the textbook) of future topics studied much more effective. The teacher should explain the notation and style of the author and indicate the author's pedagogical intentions and any other peculiarities that may be appropriate. Teachers should also help students develop the analytical skills for identifying when, why, and how a particular model described in the text fits a particular problem. Students must constantly question their understanding of each idea and look toward the overriding direction or "big picture" of the concepts or unit being developed and how they are related to other concepts. Oftentimes, mathematics textbooks offer model solutions to problems. These should also be read in a very active fashion before doing the exercises, even if the student thinks he or she can "fly" through the exercises after or without reading the explanatory material.

Precautions and Possible Pitfalls

The teacher should make a special point of instructing students to read the textbook regularly. In doing so, teachers should highlight specific aspects of the readings, such as the differences between class instruction and the textbook material (if such exists). The teacher should be aware that there may exist individual reading problems with students in the class that may not manifest themselves in their mathematics achievement. That is, a good mathematics student could be a poor reader. The teacher's awareness of and sensitivity to these weaknesses are important when considering the task of reading mathematics textbooks.

Source

Long, J. D., & Long, E. W. (1987). Enhancing student achievement through metacomprehension training. *Journal of Developmental Education, 11*(1).

STRATEGY 11: Avoid reacting emotionally when evaluating problematic situations in the classroom.

What the Research Says

An emotional reaction can prevent a teacher from objectively assessing a problematic situation. When a teacher displays a high level of emotional excitement, he or she tends to evaluate situations more negatively than is objectively appropriate. This reaction is especially common when the teacher is very sensitive to the perception that students are not reacting to the teacher's demand and when the teacher is very sensitive to the perception of the students' motor activity and body language. A cohort of 132 teachers participated in an investigation of how teachers evaluate problematic situations. The participants saw several videotapes with one-and-a-half- to four-minute sequences that represented problematic situations. After every scene teachers had to complete a questionnaire that asked for their observations, evaluation, estimation of the situation, emotion, and reaction. The extent of their emotional excitement was also requested.

Teaching to the NCTM Standards

The NCTM Professional Standards state that "the teacher of mathematics should create a healthy learning environment that fosters the development of students' mathematical power."[18] Although standards in mathematics do not specifically address behavioral issues, it is important to recognize that the comprehension and mastery of mathematics can sometimes be frustrating for the struggling student. Teachers' sensitivity to this issue can promote an atmosphere where student frustration can be channeled into productive activities that can put the student back on track. The application below highlights how a student's chronic motor activity may indicate an underlying problem in comprehension and not a behavioral problem. Teachers who are sensitive to this can provide immediate support to alleviate the frustration that accompanies student failure to comprehend mathematical concepts.

Classroom Applications

Beware of vicious cycles! Teachers frequently penalize students who show a high degree of motor activity or conspicuous body language. Students with conspicuous motor activity or pronounced body language may not be able to stop it immediately. Chronic motor activity or a student's present mood may prevent that student from controlling motor activity entirely. When a teacher gets the impression that students are not reacting to the teacher's demand to stop moving, teachers tend to evaluate the situations more negatively.

How to break the vicious cycle:

- Sanction the student.
- Be patient for a few seconds; students need time to realize just what you are asking of them. (Careful! These seconds can seem like ages from your perspective.)
- Establish eye contact. In this way the student will get the impression that you mean it.
- If it is necessary to repeat your demand, do it word for word, as originally presented. Do not confront the student with what might appear to be a different demand or request. This could confuse the issue.

Precautions and Possible Pitfalls

In giving the appearance of being simultaneously well balanced and strict, you have to work with high self-control. If you change suddenly and become very emotional, the student's behavior is likely to worsen. By maintaining your equilibrium—not reacting emotionally in problematic situations—you can prevent the cycle from starting again.

Source

Albert Thienel. Der Einfluß der emotionalen Betroffenheit von Lehrern auf das differentielle Erleben einer Problemsituation (The influence of emotional excitement of teachers by differential definitions of a problematic). *Psychologie in Erziehung und Unterricht*, 36 Jg., (1989) S. 210–215.

STRATEGY 12: Carefully select problems for use in cooperative learning groups.

What the Research Says

Cooperative learning in high school mathematics classes is often viewed by both teachers and students as having a beneficial effect on the learning environment. Both preferred small group work in comparison to traditional methods of instruction. Students became aware that the keys to learning are teaching and explaining. One study examined two teachers who used cooperative learning frequently in class; it investigated their and their students' perceptions about cooperative learning practices in a high school mathematics classroom. The researchers observed classroom activities and interviewed teachers and students. The teachers perceived cooperative learning as an asset to the learning environment, despite some of its limitations. The results showed that some teachers thought that cooperative learning could be used effectively with all math problems. However, most research indicates that careful selection of problems for use in cooperative learning groups is important.

Teaching to the NCTM Standards

The *NCTM Handbook of Research on Mathematics Teaching and Learning* devotes much attention to the effectiveness of cooperative learning. Some of the benefits cited in the research include "students' involvement in curriculum tasks increased during cooperative, small-group work."[19] However, "the opportunity provided during small-group instruction varies for different students."[20] The application below supports this notion. Problems must be selected with great care. Problems that are too challenging will not engage the lower-achieving students, who typically have the greatest difficulties adapting to the cooperative group model. The research also found "low achievers may be discouraged from active participation in tasks because the more academically competent group members are concerned with getting tasks completed as quickly as possible."[21]

Classroom Applications

One example of an activity where cooperative learning may be beneficial is to have students investigate (or solve) a problem such as the following:

> A straight railroad track passes near two towns. One town is 5 miles from the track while the other is 8 miles from the track. The two towns, which are on the same side of the track, are 18 miles apart from each other. The citizens of the two towns want to build a railroad station somewhere along the track so that it is equally accessible from the two towns. Where should the station be placed?

Discuss the kinds of considerations that might be important in the decision. Explain and justify the geometric solution to the problem. Explain which other variables (not given) might be necessary to give the best location for the station.

Precautions and Possible Pitfalls

Perceptions don't always match reality. When planning to use cooperative learning, be selective about the types of problems or tasks given to the cooperative learning groups. Research on the effectiveness of cooperative learning in mathematics indicates it works more suitably with some math problems and skills than others. However, teachers' perceptions may not reflect this selectivity, and teachers may (unadvisedly) use cooperative learning across the board. Easy problems are unlikely to work effectively in cooperative learning groups because there is less need to collaborate to help each other.

Source

Peele, A., & McCoy, L. (1994). *Perception of group work in mathematics.* Paper presented at the American Educational Research Association Annual Meeting, New Orleans.

STRATEGY 13: Encourage students to work cooperatively with other students.

What the Research Says

When students cooperate with other students they often get more out of learning than they do when working on their own or even when working with the teacher. When students are isolated from each other and compete with each other, they are less involved in learning, their learning is not as deep, and they have fewer opportunities to improve their thinking. Students who work cooperatively achieve at higher levels, persist longer when working on difficult tasks, and are more motivated to learn because learning becomes fun and meaningful.

Teaching to the NCTM Standards

The NCTM Communication Standard stresses the importance of students becoming independent learners and students learning from each other. The concept of establishing a classroom community for learning is encouraged and valued as students become stakeholders in their own instruction program. The Communication Standard states: "The instruction program should enable all students to communicate their mathematical thinking coherently and clearly to peers, teachers, and others."[22] The *Handbook of Research on Mathematics Teaching and Learning* (NCTM) suggests that students are too passive and need to become more involved intellectually in classroom activities. As the application below suggests, breaking students into small groups engages more students in the learning process and can lead to a deeper understanding by a greater number of students.

Classroom Applications

Working cooperatively in a mathematics classroom can be done in many ways. Students can be given a challenging problem and then asked to solve it in small groups. While working cooperatively in a group setting, students must verbalize their thoughts and discoveries, which helps them to understand these ideas and use them as steps on the path to a solution. Teachers may present classical challenges such as filling in the cells of a 3×3 grid to form a magic square. Or, they may simply ask open-ended questions where mathematical solutions include judgmental aspects. For example, some real-life problems of this nature could be to determine the best location for a railway station, the minimum sum distance from town to station to town, and other conditions that can be included to describe a real situation. These factors present a problem that can be solved in a variety of ways by making judgments about the relative importance of variables. Such open-ended problem situations provide excellent opportunities for students to work cooperatively, probing and prodding each other in their quest for a solution.

Precautions and Possible Pitfalls

Not all group work is cooperative learning! Students must work together, help each other, and learn from each other in order for it to really be cooperative learning. Beware that one person doesn't dominate the group, which often tends to occur. Assigning group roles is one way of either preventing this from occurring or handling it when it does. Teachers need to constantly monitor groups as they are solving problems to make sure they stay on task and are working in productive ways.

Source

Johnson, D., & Johnson, R. (1975). *Learning together and alone: Cooperation, competition, and individualization.* Englewood Cliffs, NJ: Prentice Hall.

STRATEGY 14: Use group problem solving to stimulate students to apply mathematical thinking skills.

What the Research Says

Students interacting with other students when solving problems in a group stimulate basic (cognitive) and higher-level (metacognitive) mathematical thinking skills. A study was conducted with twenty-seven average-ability seventh graders in an urban school. A total of seventy-three problem-solving behaviors were examined for each student. One week before the study, students were put into heterogeneous groups so they could become familiar with how to work in groups and with the members of their groups. Heterogeneous groups were balanced for ability, gender, race, and ethnicity. During the study itself, no time limit was given for groups to solve the assigned problem. Groups were videotaped as they solved the problem. Working in groups to solve the problem, students engaged in the following types of mathematical thinking:

- Reading the problem (basic)
- Understanding the problem (higher level)
- Analyzing the problem (higher level)
- Planning an approach to solve the problem (higher level)
- Exploring a problem-solving approach to see whether or not it works (basic and higher level)
- Implementing the plan for solving the problem (basic and higher level)
- Verifying the final solution (basic and higher level)
- Listening to and watching other students during the problem-solving process

The greatest percentage of higher-level mathematical thinking occurred while students were exploring a solution; the second greatest occurred as students were trying to understand the problem. The highest percentage of basic mathematical thinking occurred during exploration; the second highest occurred while students were reading the problem.

Teaching to the NCTM Standards

The NCTM Professional Standards for Teaching and Learning state:

> Creating an environment that supports and encourages mathematical reasoning and fosters all students' competence with, and disposition toward, mathematics should be one of the teacher's central concerns. The nature of this learning environment is shaped by the kinds of mathematical tasks and discourse in which students engage.[23]
>
> Problem solving, reasoning, and communication are processes that should pervade all mathematics instruction and should be modeled by teachers. Students should be engaged in mathematical tasks and discourse that require problem solving, reasoning, and communication.[24]

Teaching problem solving as a group activity enables members of the class to analyze, compare, contrast, examine, and test solutions. "Students should be encouraged to explain their reasoning process for reaching a given conclusion or to justify why their particular approach to a problem is appropriate."[25] The application outlined below clearly addresses the standards and provides a stimulating format for meaningful problem solving.

Classroom Applications

Carefully select problems that will stimulate use of mathematical thinking and problem solving when preparing for group activities. Observe groups as they are working and intervene only as needed. After most groups seem to have finished, call the groups back together and have them describe their problem-solving processes and answers. Call attention to the mathematical thinking and problem-solving processes they were using, both when conferring with individual groups and after all groups have been called back together. Explicitly use the eight concepts described above when discussing problem solving to increase students' awareness of how they are thinking mathematically and how they are solving problems. Don't set rigid time limits for solving the problem. Let each group work at its own pace.

Precautions and Possible Pitfalls

Not all groups will behave in the same way. Some groups will not engage in all eight of the problem-solving behavior categories described above, and those that do may engage in them in varying degrees. In addition, not all groups will be successful in solving a given problem. If one group (or more) finishes before the others, make sure you have a follow-up task ready so students can extend their thinking rather than get bored and waste time waiting for the others to finish.

Source

Artzt, A., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of group problem solving in mathematics. *Cognition and Instruction, 9*(2), 137–175.

STRATEGY 15: Use the Jigsaw Technique of cooperative learning as an interesting and effective way for students to learn.

What the Research Says

Contrary to some beliefs about cooperative learning having only social benefits, research shows that the Jigsaw method helps students learn and apply academic content as well. An experimental study was conducted with seven classes of students in Grades 7 and 8. The 141 students were separated into four experimental classes and three control classes. The experimental classes were taught with the Jigsaw Technique, while the three control classes received regular instruction through lectures. The experiment lasted about four weeks, one double lesson per week. This study examined the social, personal, and academic benefits of Jigsaw and traditional instruction. Social and personal benefits observed to result from the Jigsaw method are growth of self-control, self-management, ambition, independence, and social interaction. Jigsaw was also found to reduce intimidation in the classroom, which inhibits learning and leads to introverted student behavior. The academic benefits of Jigsaw include improved reading abilities, systematic reproduction of knowledge, ability to make conclusions, and summarizing.

Students in the Jigsaw classrooms demonstrated improved knowledge as well as their ability to apply that knowledge when compared with students in traditional classes. Students were not afraid to ask questions or to scrutinize presented information when they were able to ask for and get an explanation of something from a peer.

Teaching to the NCTM Standards

The NCTM *Handbook of Research on Mathematics Teaching and Learning* discusses "the use of small cooperative groups of heterogeneous ability."[26] "The efficacy of small cooperative groups for increasing mathematics achievement in the general population seems well established."[27] The use of the Jigsaw techniques as described below engages all members of the class and clearly supports the communication standard as students are communicating their mathematical thinking and ideas to their peers.

Classroom Applications

The Jigsaw Technique operates in six steps:

1. Separate a new part of the curriculum into five major sections.
2. Split a class of twenty-five students into five groups of five students each. These groups are the so-called *base groups*. (The groups should be heterogeneous in terms of gender, cultural background, and achievement levels.)

3. Every member of the base group selects or is assigned one of the major sections. For example, one member might focus on the section on fractions, another might focus on the section on decimals, another focus on the section on percentages, and so forth. If the number of group members exceeds the number of sections, two students can focus on the same section.

4. The base groups temporarily divide up so each student can join a new group in order to become an "expert" in her or his topic. All the students focusing on fractions will be in one group, all the students focusing on decimals will be in another group, and so on. These students work together in temporary groups called *expert groups*. There they acquire the knowledge about their topic and discuss how to teach it to students in their base groups.

5. Students return to their base groups and serve as the expert for their topics. Everyone then takes a turn teaching what he or she learned about his or her topic to members of his or her base group.

6. A written test is given to the entire class.

In Steps 4 and 5, students get an opportunity to discuss and exchange knowledge. Step 6 gives the teacher an opportunity to check the quality of students' work and to see what and how much they learned from each other. One of the advantages of this method of cooperative learning is that in Jigsaw there is always active learning going on and students do not become bored while passively listening to reports from other groups, as sometimes happens with the Johnson and Johnson "Learning Together" method.

Jigsaw can be used to teach a series of unrelated skills such as factoring, reducing or simplifying algebraic fractions, as well as topics that could be used to tie seemingly unrelated topics together, such as verbal problems.

Precautions and Possible Pitfalls

While students teach members of their base groups in Step 5, teachers are frequently tempted to join in the discussions and advise students regarding the best way to teach the subject to their base group. This type of teacher intervention prevents the social and intellectual benefits of Jigsaw. Although a teacher has to monitor groupwork in order to intervene when there are substantial mistakes in understanding the academic content, the teacher should not interfere with how students decide to teach this content to their peers.

Sources

Aronson, E., Blaney, N., Stephan, C., Sikes, J., & Snapp, M. (1978). *The Jigsaw classroom*. Beverly Hills, CA: Sage.

Renate Eppler & Günter L. Huber. Wissenswert in Team: Empirische Untersuchung von Effeckten des Gruppen-Puzzles (Acquisition of knowledge in teams: An empirical study of effects of the Jigsaw-techniques. *Psychologie in Erziehung und Unterricht, 37* Jg., (1990) S. 172–178.