

MATHEMATICAL TASKS AND TALK THAT GUIDE LEARNING

3



Ms. Clark was planning a lesson on counting the value of coins for her first graders. Her learning intention for the lesson was for students to determine the value of up to four coins including pennies, nickels, and dimes. Her success criterion was for students to successfully apply their understanding to a new situation. She considered the work in the first-grade text that included drawings of several coins of which students were to determine the total value. Since they had been spending a lot of time on this skill, she was certain this would not be very challenging for her students. Instead, she decided to give them the following task.

You are going to the store and you want to buy a banana that costs 25¢. You have lots of pennies, nickels, and dimes. What coins can you use to pay for the banana?

Ms. Clark brought a variety of coins to class so that each group had a selection of coins to help them with the problem. She was surprised at the reluctance of the students to get started on what she thought would be an enjoyable task. It turned out some students didn't recognize the real coins (even though they recognized the drawings in the textbook). Other students recognized the coins but had no idea of how to put them together to make 25¢. Ms. Clark did not jump right in to tell the students what to do. Instead, she encouraged them to work in groups to support each other in solving the problem. She was intrigued to watch the groups form based on what students could do (recognize or count the coins). Soon, one group of students raised their hands to show their answer of two dimes and a nickel. When Ms. Clark asked if there was another way to make 25¢, the students were dumbfounded. They had never solved a problem with more than one correct answer! Interestingly, the students set to work to find other solutions, challenging themselves to find all of the possible combinations!

Making Learning Visible Through Appropriate Mathematical Tasks

The banana problem is an example of students having surface learning (recognizing coins and/or knowing the value of individual coins) and taking that learning to a more complex level through deep learning

(combining the value of various coins) to transfer learning. Not only did they have to recognize and add the value of the coins, but unlike the textbook exercises, they also had to determine which coins to use. Giving students appropriate tasks at the right time in their learning cycle is crucial to move students from surface to deep and transfer learning.

Exercises Versus Problems

It is important to have a common understanding of the types of tasks we assign to students. **Exercises**, which typically make up most of traditional textbook practice, are provided for students to practice a particular skill, usually devoid of any context. Although these are casually referred to as problems, in reality they are simply practice exercises.

Problems have contexts—they are usually written in words that can be situations that apply or provide a context for a mathematical concept. One category of problems is an application that focuses on the use of particular concepts or procedures. Another category of problems is non-routine or open-ended problems that involve much more than applying a concept or procedure. We will explore each of these types of problems in more detail in the next section.

There are a few items that we need to address before we more fully explore the types of tasks that are useful in various phases of learning in mathematics.

- Spaced practice—also known as distributed practice—is much more effective than mass practice. We will discuss this more in Chapter 4. In practical terms, this means that students should do a few exercises or problems on a given concept each day over several days rather than a lot of problems for only one or two days.
- Math is not a speed race. Teachers should be very careful with timed tests. Neither fluency nor stamina requires that students work as quickly as humanly possible. Giving students a test that requires them to speed through problems reinforces an idea that they should prioritize by doing the “easy” problems first and not spend valuable time on problems that require deeper thinking. Too often, timed tests or speed games are used to check for fluency with basic mathematics facts. The problem is, speed is not part of fluency. Fluency requires flexible, accurate, and efficient

Exercises are meant to practice a particular skill, but are noncontextual.

Problems are usually written in words that can be situations or provide a context for a mathematical concept.

EFFECT SIZE FOR
SPACED VERSUS MASS
PRACTICE = 0.71

thinking. Fluency also requires a level of conceptual understanding. One would not be considered fluent in a foreign language if he or she could speak it by mimicking without any comprehension! Students would be better served with practice developing fluency rather than racing through written tasks or activities. In addition, speed races also make some students believe that they are not good at math. The attitude students have toward mathematics is important and can impact their willingness to try.

- Tasks should not focus exclusively on procedures. Sara excelled in math at a young age. She seemed to understand numbers, and she was very good at learning a procedure and executing it repeatedly on her own. But she was never asked to explain why these procedures worked. Bring down the last number under the house when doing long division? Sure, why not. Why does that lead to a correct answer? How do I apply that skill to real-world situations if I don't understand what it means? Sara had no idea, and it didn't seem to matter to her teacher. This was a case of focusing on procedural skills and sacrificing conceptual understanding.

We are not arguing that students shouldn't learn long division. But we don't think that students gain much from doing long division mindlessly, either. The goal should be for students to develop a transferable and flexible understanding of processes like division, and they should have the opportunity to construct this understanding in a meaningful context. Doing extensive, repeated, context-free long division exercises is just not aligned to this goal.

Instead, students should be expected to engage in reasoning, exploration, flexible thinking, and making connections. They know that learning isn't easy, and they should enjoy the success of meeting the challenges that learning demands of them (Hattie, 2012). Students need deliberate practice, guided by the teacher, not repetitive skill-and-drill tasks. Some tasks should provide students an opportunity to engage in mathematical modeling—taking a problem or situation, representing it mathematically, and doing the mathematics to arrive at a sensible solution or to glean new information that wouldn't have been possible without the mathematics.

Still other tasks require that students practice applying a concept in different situations. To facilitate strategic thinking, some tasks should be open-ended and have multiple paths to get to the solution or, in some

cases, solutions. Math tasks don't always have to be fun, but they can be interesting and useful.

Should students work on exercise sets, in which they develop skill in long division? Sure, but these types of tasks won't be discussed here for several reasons. First, we have seen that teachers are already quite good at assigning exercises from a textbook, and reading about this would be a waste of your time. More importantly, though, the research evidence suggests that application of a concept, in varying contexts or in ways that offer sense-making opportunities, is more effective in building true fluency than doing repeated, nearly identical manipulations of numbers (NCTM, 2014).

It is useful for students to be able to perform math operations flexibly and efficiently, as it frees up cognitive space to apply these operations to novel situations and relate these operations to other mathematics concepts. But in most mathematics classes, this type of automaticity tends to be emphasized way too early in the learning cycle. It also tends to take up a disproportionate amount of class time. Procedural fluency cannot be developed without true and meaningful comprehension, and “drill-and-kill” exercises without understanding can harm students' mathematical understandings, their motivation level, and the way they view mathematics. Students who learn procedures at the expense of mathematical thinking often fail to develop an understanding of what they're doing conceptually, and teachers find that it's more difficult to motivate students to really understand a concept if they can already execute a shortcut. What's needed is a restoration of the balance: A strong conceptual foundation makes fluency building more efficient, meaningful, and useful for students. So it really is worth devoting a lot more learning time to the conceptual understanding that undergirds procedural knowledge. Children need to learn the relationship between procedures and concepts in order to become increasingly fluent thinkers.

Problems fall into two categories: applications and nonroutine problems. **Applications**—often called word problems or story problems—are problems, usually related to real-life experiences, in which students use or apply a mathematical concept or skill they have learned. Interestingly, these problems usually follow the exercises in a traditional textbook lesson. However, they should also be used to introduce an idea in order to allow students to model a situation and develop conceptual understanding, connect that understanding to procedural skills, and then practice that skill through more applications and exercises. For those familiar

Procedural fluency cannot be developed without true and meaningful comprehension, and 'drill-and-kill' exercises without understanding can harm students' mathematical understandings, their motivation level, and the way they view mathematics.

Applications are problems, usually related to real-life experiences, in which students use or apply a mathematical concept or skill they have learned.

with Cognitively Guided Instruction (Carpenter et al., 2014), this is the pathway used in that philosophy. Application problems can range from straightforward (solution reached by applying well-practiced operations) to difficult (involving application of new ideas, several steps, and/or multiple representations).

Non-routine or complex problems are problems that involve more than applying a mathematical procedure for solution.

Non-routine or complex problems are problems that involve more than applying a mathematical procedure for solution. These types of problems are usually met with student reactions of “I don’t know what to do!” because a simple procedure is not the pathway to a solution. Rather, students need to use a variety of strategies and some “out-of-the-box” thinking to solve these types of problems.

When we think about the kinds of mathematical tasks we want to use with our students, and when we should use each kind of task (and there is a place in mathematics instruction for each type of task), we need to think about what we want to achieve with the task. What are our learning intentions? What role does the task play in helping students meet the success criteria for the lesson?

In the next sections, we will examine two frameworks for classifying problems. One focuses on the level of difficulty/complexity of the task, and the other focuses on the kind of thinking required by the student. One is not better than the other, but given your own realm of experience, one may be more helpful than the other as you work to connect exercises/problems with surface, deep, and transfer learning. We will go into more detail with examples in future chapters. Our intention here is to get you familiar with the descriptions and the need for hard thinking about the kinds of tasks you assign to your students to make your teaching positively impact student learning.

Difficulty is the amount of effort or work one must put in.

Complexity is the level of thinking, the number of steps, or the abstractness of the task.

Difficulty Versus Complexity

In order to help students master all dimensions of rigor (conceptual understanding, procedural fluency, and applications) and to help students’ progress toward owning their own learning and then transferring that learning to new situations, it is important for teachers to think carefully about the level and type of challenge a given task provides. Unfortunately, some people confuse difficulty with complexity. We think of **difficulty** as the amount of effort or work a student is expected to put forth, whereas **complexity** is the level of thinking,

DIFFICULTY AND COMPLEXITY



Figure 3.1

the number of steps, or the abstractness of the task. We don't believe that teachers can radically impact student learning by simply increasing the volume of work. We know that students learn more when they are engaged in deeper thinking. Figure 3.1 shows how we think of this in four quadrants.

The fluency quadrant that includes tasks of low difficulty and low complexity is not unimportant; it's where automaticity resides. For example, once students have mastered conceptual understanding of addition and subtraction (what do they mean and what do they look like?) and learned thinking strategies and procedures for computing sums and differences, they need to build fluency so that they are flexible, accurate, and efficient with these operations. Students should be able to do basic mathematical calculations quickly and effortlessly in order to free up the cognitive space to connect the operations to more complex examples or to larger concepts. There are times when you will want students to build automaticity on certain types of procedures. Instant retrieval of

basic number facts is foundational for being able to think conceptually about more complex mathematical tasks. Hattie and Yates (2014) assert that these retrievals are the product of



Video 3.1
What We Mean
by Tasks With Rigor

[http://resources.corwin.com/
VL-mathematics](http://resources.corwin.com/VL-mathematics)

a combination of exposure to others, working it out for yourself, playing with concrete materials, experimenting with different forms of representation, and then rehearsing the acquired knowledge unit within your immediate memory, transferring it into long-term memory, and having it validated thousands of times. (p. 57)

If students' mathematical experiences are limited to this quadrant, learning isn't going to be robust. The stamina quadrant—high difficulty but low complexity—is where tasks that build perseverance reside. Stamina refers to the idea of sticking with a problem or task even when the work is difficult and requires patience and tenacity. This type of task would be a problem or exercise (yes, they both have a place here) in which students are taking their current knowledge and extending it to a more difficult situation. The first-grade banana task that opened this chapter is a good example of a task that promotes stamina. Students were able to complete earlier work with counting coins in the textbook examples, but they needed to apply this knowledge differently and think strategically about the different ways to find *all* of the possible solutions, and then justify how they knew they had them all.

The daily practice of having students work independently to resolve a problem before consulting peers is one example of helping to build stamina, as it draws on the learner's capacity to stick with a problem. Add to that the additional step of consulting one another and then returning to the problem individually a second time to make any corrections, and now you're extending their stamina even further.

The strategic thinking quadrant addresses tasks that have a lower level of difficulty, but a higher degree of complexity. Some rich mathematical tasks fall into this category, as they draw on students' ability to think strategically. An example of strategic thinking is having students connect their understanding of division of whole numbers to division of decimals before any specific procedure is explored. In this task, students must think about what they know about division and what they know about decimals to make conjectures about place value in the quotient.

Mr. Beams has a very strange calculator. It works just fine until he presses the = button. The decimal point doesn't appear in the answer. Use what you know about decimals and division to help him determine where the decimal point belongs in each quotient. Be ready to justify your thinking!

1. $68.64 \div 4.4 = 156$

2. $400.14 \div 85.5 = 468$

3. $0.735 \div 0.7 = 105$

4. $51.1875 \div 1.05 = 4875$

This task requires students to extend their understanding from previous learning to situations that are much more complex. Complexity is often supported by having students work in groups and justify their thinking. Students will likely be stretched to consider how to resolve problems collaboratively, attend to group communication and planning, and monitor their own thinking and understanding.

The final quadrant, which describes expertise, includes those tasks that are both complex and difficult. These tasks, in one form or another, push students to stretch and extend their learning. A favorite task for fifth or sixth graders is the Handshake Problem, which includes both complexity and difficulty.

Twenty-five people attended a party. If each person shakes hands with every other person at the party, how many handshakes will there be?

This problem can be pretty overwhelming as there is not a particular process or operation that will lead to a solution. Rather, students might work together to use a combination of problem-solving strategies to get started, including acting it out, looking for a pattern, making a table, or starting with a simpler problem. What makes this problem even more interesting (and complex) is the opportunity for students to make a generalization (find a rule) so they can determine the number of handshakes for any number of guests—even 1,000!

This is certainly not an exhaustive list; rather, it is meant to be illustrative. As part of each lesson, teachers should know the level of difficulty and complexity they are expecting of students. They can then make decisions about differentiation and instructional support, as well as feedback that will move learning forward.

Students need regular contact with tasks that allow them to explore, resolve problems, and notice their own thinking. They need tasks that present the right amount of challenge relative to their current performance and understanding, and to the success criteria deriving from the learning intention. Teachers should select tasks that help students push their thinking, but are not so difficult that the learner sees the goal as unattainable. Teachers *and* students must be able to see a pathway to attaining the goal. This supports the second effective teaching practice in NCTM's *Principles to Actions*: Implement tasks that promote reasoning and problem solving. The tasks that teachers assign must

1. Align with the learning intention.
2. Provide students an opportunity to engage in exploration and make sense of important mathematics.
3. Encourage students to use procedures in ways that are connected to understanding.
4. Provide students opportunities to implement the standards for mathematical practice.
5. Allow teachers and students to determine if the success criteria have been met.

This is why relating a task to prior learning is so important (Hattie, 2012).

A Taxonomy of Tasks Based on Cognitive Demand

A second framework for thinking about how to strategically select mathematical tasks aligned to learning intentions and success criteria is one that presents a taxonomy of mathematical tasks based on the level of cognitive demand each requires (Smith & Stein, 1998). **Cognitive demand** is the kind and level of thinking required of students in order to successfully engage with and solve the task (Stein, Smith, Henningsen, & Silver, 2000).

This taxonomy has been embraced by the National Council of Teachers of Mathematics (NCTM, 2014) for good reason, as it provides a powerful

Cognitive demand is the kind and level of thinking required of students in order to successfully engage with and solve a task.

CHARACTERISTICS OF MATHEMATICAL TASKS AT FOUR LEVELS OF COGNITIVE DEMAND

Levels of Demands

Lower-Level Demands (Memorization)

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous; such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-Level Demands (Procedures Without Connections)

- Are algorithmic; use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task
- Require limited cognitive demand for successful completion; little ambiguity exists about what needs to be done and how to do it
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-Level Demands (Procedures With Connections)

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations; making connections among multiple representations helps develop meaning
- Require some degree of cognitive effort; although general procedures may be followed, they cannot be followed mindlessly—students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding

(Continued)

(Continued)

Higher-Level Demands (Doing Mathematics)

- Require complex and non-algorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

Source: Smith and Stein (1998). Used with permission.

Note: These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM, 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen, 1996; Stein, Lane, and Silver, 1996).

Figure 3.2

structure to types and characteristics of mathematical tasks, providing teachers with criteria that enable them to align the type of task they choose with the learning intention and success criteria for a given outcome (see Figure 3.2).

Traditionally, the majority of classroom instructional time is spent on tasks with lower level cognitive demands that require memorization and/or procedures without connections. These are not bad tasks, and there is a time and place for them, but they do not provide students the range of learning experiences they need to develop mathematical habits of mind, such as looking for patterns and using alternate representations (Levasseur & Cuoco, 2003). Memorization tasks that follow the development of conceptual understanding facilitate learning at the surface level. And surface learning is important and should not be minimized. There has been much misdirected criticism of surface learning because it is often confused with shallow learning. That said, too much emphasis on surface learning at the expense of learning that deepens over time

and transfers to new and novel situations does not provide students with true mathematical experiences. Balance is warranted.

Tasks with higher levels of cognitive demand on Smith and Stein's taxonomy—those that connect procedures to understanding—require students to understand relationships between concepts and processes as they analyze and explore the task and its parameters. But the process doesn't stop there. Tasks that call for higher level cognitive demand extend even further to those requiring more complex thinking. There is no predictable or well-rehearsed pathway (algorithm) that is suggested by the task, or by a similar and already-worked example. Tasks such as these provide students an opportunity to engage transfer learning.

However, effective teachers don't leave these things to chance. Instead, they provide problem-solving experiences in which students engage with rich tasks that require them to mobilize their knowledge and skills in new ways. A close association between a previously learned task and a novel situation is necessary for promoting transfer of learning. In time, these become tasks that stretch students' problem-solving abilities as they self-monitor and self-regulate their learning. This is transfer learning in action.


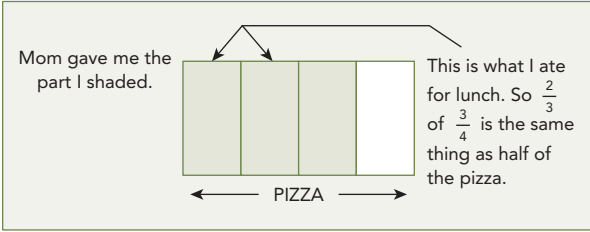
EFFECT SIZE FOR
PROBLEM-SOLVING
TEACHING = 0.61

Whether you are looking at a task in terms of difficulty versus complexity or the level of cognitive demand students must employ, appropriately challenging tasks may produce some level of student anxiety when they are first introduced. As we have noted before, that's okay, because students should expect learning to require an effort as they grow to appreciate cognitively demanding tasks. An often-needed requirement for learning to occur is some form of tension, some realization of "not knowing," a commitment to want to know and understand—or, as Piaget called it, some "state of disequilibrium" (Hattie, 2012). When students are assigned rich tasks, they use a variety of skills and ask themselves questions, make meaning of mathematics, and ultimately build a healthy and realistic relationship to mathematics as something that is engaging, interesting, and useful—and something that makes sense.

Figure 3.3 includes examples of mathematical tasks for each level of cognitive demand.

We will refer back to these tasks and present additional tasks for your consideration in the coming chapters. In the meantime, we encourage you to sharpen your pencils and experience the levels of cognitive demand along with some metacognition by completing these tasks. Note that answers are not provided in the back of this book!

EXAMPLES OF TASKS AT EACH OF THE FOUR LEVELS OF COGNITIVE DEMAND

Lower-Level Demands <i>Memorization</i>	Higher-Level Demands <i>Procedures With Connections</i>
<p>What is the rule for multiplying fractions?</p> <p>Expected student response:</p>	<p>Using pattern blocks, if two hexagons are considered to be one whole, find $\frac{1}{6}$ of $\frac{1}{2}$. Draw your answer and explain your solution.</p> <p>Expected student response:</p>
<p>You multiply the numerator times the numerator and the denominator times the denominator.</p>	
<p>or</p>	<p>First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So $\frac{1}{6}$ of $\frac{1}{2}$ is $\frac{1}{12}$.</p>
<p>You multiply the two top numbers and then the two bottom numbers.</p>	<p><i>Doing Mathematics</i></p> <p>Create a real-world situation for the following problem:</p>
<p><i>Procedures Without Connections</i></p> <p>Multiply:</p>	$\frac{2}{3} \times \frac{3}{4}$
$\frac{5}{6} \times \frac{7}{8}$	<p>Solve the problem you have created without using the rule, and explain your solution.</p> <p>One possible student response:</p>
$\frac{4}{9} \times \frac{3}{5}$	<p>For lunch Mom gave me three-fourths of the pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat?</p>
<p>Expected student response:</p>	<p>I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be one two of the shaded sections.</p>
$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$ $\frac{5}{6} \times \frac{7}{8} = \frac{5 \times 7}{6 \times 8} = \frac{35}{48}$ $\frac{4}{9} \times \frac{3}{5} = \frac{4 \times 3}{9 \times 5} = \frac{12}{45}$	

Source: Smith and Stein (1998). Used with permission.

Figure 3.3

Making Learning Visible Through Mathematical Talk

We're mindful that these tasks don't exist in a vacuum. These meaningful tasks are fueled by the discourse that occurs in productive class conversations. The language, thinking, and reasoning that occur when discourse happens further contribute to surface, deep, and transfer learning. Discourse is facilitated through purposeful questioning and thoughtful prompts and cues that usually begin with the teacher. Just as there is a need to select tasks that align with learning intentions and success criteria, there are a variety of "math talk" routines and techniques teachers can use to build student understanding and assess how that understanding is developing, and to guide students in self-questioning and self-verbalization to extend metacognition. As teachers consider routines and techniques that facilitate rich classroom discourse, they should also be thinking about the role of discourse in supporting surface, deep, and transfer learning.

EFFECT SIZE
FOR SELF-
VERBALIZATION
AND SELF-
QUESTIONING
= 0.64

Characteristics of Rich Classroom Discourse

Let's begin by examining the characteristics of classroom discourse that builds student understanding and confidence.

1. Teacher questioning and prompts support students in building understanding based on previous knowledge and making connections rather than the teacher being the authority.
2. Mistakes are valued and seen as opportunities for students to clarify their ideas through discussing and justifying their thinking and listening to the ideas of their peers.
3. Students consider different approaches to the mathematics and how those approaches are similar or different.
4. There is an element of productive struggle among students that is accompanied by perseverance, so that the focus is on how students are going to use mathematics to make sense of the task and how to approach a solution path.
5. Students are encouraged to use a variety of representations to build understanding and justify their thinking.

So, how do we facilitate this kind of rich discourse in our classroom? Read on!

Posing Purposeful Questions

Let's begin by delving into questioning techniques that have different purposes and goals throughout a lesson. In subsequent chapters, we will refer back to these general categories of questions and offer concrete examples of when in the learning cycle these techniques are most appropriate. Purposeful questions serve a variety of outcomes (NCTM, 2014), including the following:



Video 3.2
Questioning That
Guides Learning

[http://resources.corwin.com/
VL-mathematics](http://resources.corwin.com/VL-mathematics)

- Encouraging students to explain, elaborate, and clarify thinking to build understanding
- Revealing students' current understanding of a concept
- Making the learning of mathematics more visible and accessible for students

By the way, these are not discrete outcomes. It is likely that questions that are intended to support students in building understanding and applying current knowledge to new situations are also providing the teachers information about students' current understanding of a concept. And of course, making the learning of mathematics more visible and accessible for students is the overarching goal for all of our work in recognizing and applying impactful instruction.

Questions That Check, Build, and Deepen Student Understanding

Have you ever started a lesson that builds on previous understandings only to find the students seem to be in the Twilight Zone? Mrs. Norton recalls a situation just like that. After teaching fifth-grade mathematics for many years, she was "promoted" with her students to teach sixth-grade mathematics the following year. When they were ready to extend their understanding of fractional numbers, she thought that there wouldn't be much need for review. After all, fifth grade was the "year of the fraction," and she knew her students had developed understanding through the use of concrete explorations and a variety of applications. So she began the first lesson asking students to solve this example and be ready to explain their thinking:

$$\frac{2}{3} + \frac{1}{2}$$

Imagine her surprise when every student had the answer $\frac{3}{5}$. After she calmed down a bit, she asked her students, "How would you convince

me your answer is correct?" Interesting discourse about the sum and whether it was reasonable began to take place. Students began to think about the value of each addend in reference to $\frac{1}{2}$ and one whole rather than a procedure that made no sense. They came to the conclusion that the answer had to be greater than one and that $\frac{3}{5}$ was only a little more than $\frac{1}{2}$; therefore it wasn't reasonable. Several things happened in this lesson. First, and foremost, Mrs. Norton realized that even though she had progressed through the "steps" of concrete, pictorial, and abstract representations in teaching how to add fractions, her students had developed no number sense about fractions. She also realized how powerful the question she asked was for the students so they could begin by taking time to think about what these fractions meant and determine why their answer was not sensible. Subsequent review and lessons built on fraction benchmarks helped to develop deeper understanding of this important concept. In this lesson, Mrs. Norton valued students thinking through the questions she posed. Subsequent discourse provided her with information about her students' misconceptions and, at the same time, pushed student thinking forward.

Purposeful questions promote understanding that can be surface, deep, or transfer depending on where students are in the spiral. Rather than telling students what to do, good questions will move student thinking forward, possibly causing some disequilibrium along the way, so students can work to build on what they know and how to make sense of a given example or problem context. These questions are used not only to prompt student thinking but also to help students explain and justify their thinking. Let's revisit the decimal examples from earlier in the chapter. Good questions will help to promote student understanding even when a concept is new. As students discuss their thinking about where the decimal point belongs in the quotient of $0.735 \div 0.7 = 105$, Mr. Beams's questions require them to think more deeply about what is actually happening with the numbers.

Marcus: I think the decimal point belongs before the 1.

Mr. Beams: How many of you agree with Marcus? (*Pause.*) Marcus, can you explain why you think the decimal point belongs there?

Marcus: Well, both of the numbers I am dividing have the decimal point before the first number, so the decimal point should also be before the first number in the answer.

Mr. Beams: What do the rest of you think about Marcus's reasoning? *(Some nods and other hands are waving to get Mr. Beams's attention.)* What are some other ideas that you have?

Lisa: I think the decimal point belongs between the one and the zero because this problem means how many groups of seven tenths can I make from 735 thousandths.

Bill: But I don't get what you just said. How do you find how many tenths are in thousandths? You didn't convince me that 1.05 is the correct answer.

(A long pause takes place and quiet conversations are happening around the room. Mr. Beams lets this go on for a while and then reconvenes the class by asking the following question.)

Mr. Beams: Can anyone answer Bill's question? How can you explain how many tenths are in 0.735?

Martina: If you look at the place value of 0.735, you see that there is a seven in the tenths place. That means I have seven tenths and some more in that decimal number. If I want to know how many groups of 0.7 I can make, I can determine there is one group of 0.7 and I have a little left to make part of another group. So the decimal belongs after the 1.

Mr. Beams: Can anyone repeat what Martina just said?

Mr. Beams: What does 1.05 mean?

Patricio: It means there is one group of seven tenths in 1.05 and part of another group.

A lot is going on here. Notice that Mr. Beams never tells the students what to do. Marcus's response to the task tells Mr. Beams that some students are looking for a procedure that is neither accurate nor based on mathematical understanding. Mr. Beams allows students some time to talk to each other to make sense of the situation. His questioning carefully draws students back in and allows them to make sense of the example and think of it in terms of previous understandings of division.

While questions that check for understanding are a crucial way to guide learning, the best teachers probe further for more specific information. They don't just want to know whether or not a student understands

Teaching Takeaway

Use questions, prompts, and cues to both help children deepen understanding and also better understand student misconceptions or partial understanding.

something; they want to see if the child can explain his or her thinking and apply what is understood, or in this case, misunderstood. If a student doesn't understand, good questions enable teachers to probe deeper in order to find the point at which a misconception, overgeneralization, or partial understanding led students astray. In the back of the teacher's mind is the question "What does this child's answer tell me about what he or she knows and doesn't know?" This is followed by "What question should I ask next?" This is what helps the student begin to move from surface to deep learning.

Funneling and Focusing Questions

You might agree that formal evaluation tools like rubrics are great for longer term, mathematically rich tasks and projects, but it's important to have methods of checking for understanding that you can do anytime you like, regardless of the task. Teachers need to know how much students have actually learned, and how successful a lesson is, in real time so that they can make midcourse adjustments and differentiations. The tools teachers rely on most are the questions we ask of our students. But too often, the questions we pose are interrogative rather than invitational. By this we mean that questions that constrain student responses to short replies are not going to yield much information to the teacher. In addition, these narrow questions don't do much to provoke thinking in students, or to help them notice their own learning. Herbel-Eisenmann and Breyfogle (2005) distinguished between two patterns of teacher-student interactions: funneling questions and focusing questions. **Funneling questions** (Wood, 1998) occur when a teacher guides a student down the teacher's path to find the answer. In these situations, the teacher is doing the cognitive work. **Focusing questions** support students doing the cognitive work of learning by helping to push their thinking forward.

In the book *Principles to Actions*, the National Council of Teachers of Mathematics (NCTM) makes clear the difference between funneling questions and focusing questions. Funneling questions limit student thinking by hinting at an answer, and take the thinking away from the students. Focusing questions encourage students to figure things out for themselves. "What are the measures of central tendency we can use with these data? What are mean, median, and mode?" would be funneling questions, while "What can you tell from the data?" would be a focusing question.

Funneling questions

guide students down the teacher's path to find the answer.

Focusing questions

allow students to do the cognitive work of learning by helping to push their thinking forward.

EFFECT SIZE FOR
QUESTIONING = 0.48

Funneling Questions. Consider how little information is revealed in the following exchange, reported by Herbel-Eisenmann and Breyfogle (2005) as an example of a funneling questioning pattern:

Teacher: (0,0) and (4,1) [are two points on the line in graph B]. Great. What's the slope? (*Long pause—no response from students.*)

Teacher: What's the rise? You're going from 0 on the y [axis] up to 1? What's the rise?

Students: 1

Teacher: 1. What's the run? You're going from 0 to 4 on the x [axis].

Students: 4.

Teacher: So the slope is _____?

Students: 0.25 (*in unison with the teacher*).

Teacher: And the y -intercept is?

Students: 0.

Teacher: So $y = \frac{1}{4}x$? Or $y = 0.25x$ would be your equation. (p. 485)

Funneling questions can create the illusion of deep student learning, but really, they only require the student to know how to respond to the teacher's questioning pattern without understanding the mathematics. These types of questions limit student thinking and leave little opportunity for metacognition. This routine could also be interpreted as scaffolding. But it isn't really, since the questions direct students to what to do rather than giving them opportunities to think about and make connections in ways that effective scaffolding provides. Although the teacher is checking for understanding, the information she gets from her students is limited to whether they are correct or incorrect and doesn't consider anything about understanding or transfer of that understanding.

There can be a role for carefully thought out funneling questions as a new topic is introduced, which has greater impact than a teacher just giving procedural steps to follow. We will talk more about this as we consider surface learning strategies in Chapter 4.

Focusing Questions. The second type of questioning pattern the researchers discuss is called a focusing questioning pattern. These

questions are designed to advance student learning, not simply assess it. These are the types of questions you want to ask. Here is the beginning of the same sequence, but this time the teacher goes into a *focusing question sequence* instead of a funneling (Herbel-Eisenmann & Breyfogle, 2005):

Teacher: (0,0) and (4,1) [are two points on the line in graph B]. Great. What's the slope? (*Long pause—no response from students.*)

Teacher: What do you think of when I say slope?

Student 1: The angle of the line.

Teacher: What do you mean by the angle of the line?

Student 1: What angle it sits at compared to the x - and y -axis.

Teacher: (*Pause for students to think.*) What do you think [student 1] means?

Student 2: I see what [student 1] is saying, sort of like when we measured the steps in the cafeteria and the steps that go up to the music room—each set of steps went up at a different angle. (p. 487)

As the conversation progressed, the students engaged in figuring out how to find the slope. Students who do this are much more likely to understand slope and remember what they figured out a week later, and are much better able to transfer their knowledge—in this case, how to find the slope—to new situations, like projecting sales for a company, constructing a skateboard ramp, or learning how to find derivatives in calculus class.

You've heard the adage that "great teachers don't tell you what to see, but they show you where to look." Focusing questions open up kids' thinking and show them where to look, while funneling questions narrow their thinking in a direction that the teacher has already decided; they tell them what to see. Funneling questions don't allow for multiple paths to solving a problem, for new approaches, or for students to think about their own thinking. With focusing questions, children get to figure it out, so they learn more. They remember the content better, and they can transfer and apply it to new situations. Figure 3.4 contains examples of how funneling questions in mathematics can be transformed into focusing questions.

FUNNELING AND FOCUSING QUESTIONS IN MATHEMATICS

Funneling Questions	Focusing Questions
How do you find the mean of the data? What about the median and the mode? What about the interquartile range?	What do you notice about the data? How would you describe them to someone? What makes you say that? What other ways might you be able to describe them?
How can I get rid of the 2? What do I have to do to the other side? What about the 4?	What do you think about when you see this equation? How do you want to solve it?
How do I find the area of this trapezoid? Do you see the rectangle and the triangles? I can just add them up. How can I find the area of the rectangle?	I want to know the area of this trapezoid, but I'm not sure how to find it. Any ideas? Where should we start?
Let's add these fractions by finding the least common denominator. What's the first step in finding the least common denominator?	What should we do with these fractions? [<i>Student: "Add them."</i>] Why add them? [<i>Student refers to word problem.</i>] Okay, so how would you add them?

Figure 3.4

Some other useful focusing questions to have in your back pocket are the following:

- What are you trying to find?
- How did you get that?
- Why does that work?
- Is there another way you can represent that idea?
- How is this connected to (other idea, concept, finding, or learning intention)?

Questions that check for understanding are a crucial aspect of visible learning. The best teachers probe deeper for more specific information. They don't just want to know whether or not a student understands something. If the student does, they want to see if the child can explain his or her thinking and apply what is understood. If the student doesn't understand, these teachers probe deeper to find the point at which a misconception, overgeneralization, or partial understanding led them

astray. In the back of the teacher's mind is this question: "What does this child's answer tell me about what he or she knows and doesn't know?" This allows the teacher to determine the type of learning that the student needs next.

A key to effective checking for understanding is to avoid false positives. In other words, you don't want to fool yourself into believing that your students know something when they really don't. Novice teachers often ask a question, wait for a volunteer to respond, and then think the class gets it because the volunteer has the correct answer. This pattern doesn't work very well to get all students learning. The teachers who rely primarily on volunteers are almost always disappointed when more accurate data prove that the majority of their students haven't learned as much as their handful of volunteers. This is one advantage of having students work in groups. As you walk around and listen to group conversations, pausing to ask probing questions can provide information about where students are in their understanding rather than where one student is.

Here's one last important hint about asking good questions: The types of questions we are calling for are likely not the questions that we experienced from teachers when we were students. It takes thoughtful planning to prepare the kinds of questions that will best support your students' learning while making them more independent learners. Asking good questions models for students the kinds of questions they can ask themselves when they are stuck. Good questions are seldom spontaneous. As you are putting the practice of posing purposeful questions (NCTM, 2014) into action, give yourself time to stop and think about what question you want to ask that serves student learning and fuels constructive communication.

Prompts and Cues

Questions are the starting place that helps teachers check for understanding. Prompts encourage students to do cognitive or metacognitive work. They can take the form of a statement or a question. When Daniel Castillo said to a student who was stuck, "Based on what you know about functions, can that be true?" he wasn't just checking for understanding. He was asking the student to return to her background knowledge. Prompts should challenge students rather than do the thinking for them. **Prompts** are often used to activate background knowledge and interrupt the temporary forgetting of prior knowledge in the face of new learning. Saying "Think about what you already know about

Teaching Takeaway

Use questions to better understand student misconceptions or partial understanding.

Prompts are questions or statements used to remind students to leverage what they already know in order to think further.

finding a common denominator as you read that question again” can remind them to use what they do know. Prompts are a bit narrower than questions, as they come after you’ve had a chance to engage with the child using those focusing questions. When questions don’t spur action, prompts can move students forward.

Another prompt is revoicing what the student has said to give all students a chance to think about it, clarify whether you have understood the explanation accurately, and give the student talking an opportunity to think about his or her thinking. For example, “So you’re saying that we’ll have three-eighths left over?” This can be especially powerful if a student’s thinking seems unclear, or if he or she spoke in a way that makes it tough for other students to hear (Chapin, O’Connor, & Anderson, 2009). Another move is to ask another student, “Can you say what [student name] just said, in your own words?” This is especially helpful for English learners, and it helps the rest of the group to process what the first student said. Figure 3.5 includes sample prompts with examples.

Cues are more overt attempts to draw attention to relevant information or a certain action needed to move forward.

Cues are more direct and overt than prompts, as they shift the student’s attention to the relevant information or study action needed to move forward. Examples of effective cues are when a teacher points to a vocabulary word posted on the wall, to the lesson’s learning intentions, to another student who is using her notes, to a figure in the textbook, or to sentence starters on a table tent. If a student is looking at a page that’s different from what you assigned, then a verbal cue might be in order, such as a whispered “The class is on the other side of this paper,” or even better, “Look around at everyone else’s paper.” This doesn’t take away children’s thinking if they’ve already shown that they’re proficient in turning to the right page, since it was probably an error of whoever passed out the papers. Just like questioning patterns, you could imagine funneling cues—“Look at the left side of data table #3 when you’re deciding which numbers to use”—and focusing cues—“Think about how you could know which numbers to use” or “Remember that you have resources here to help you.” Figure 3.6 includes definitions of several types of cues.

Teaching Takeaway
Use prompts and cues to help students zero in on new learning, remember critical points, and connect to previous learning.

In using prompts and cues, teachers must be careful that they ask all students to think about why their work is correct or incorrect. Teachers can inadvertently create a situation where students know their answer is incorrect because the teacher uses certain prompts or cues that he or she does not use when seeing a correct answer. The prompt “Does that

TYPES OF PROMPTS FOR MATHEMATICS

Type of Prompt	Definition	Example
Background knowledge	Reference to content that the student already knows, has been taught, or has experienced but has temporarily forgotten or is not applying correctly.	<ul style="list-style-type: none"> When trying to solve a right-triangle problem, the teacher says, "What do you recall about the degrees inside a triangle?" As part of their study of solid figures, the teacher says, "Think about what you remember about vertices, edges, and faces."
Process or procedure	Reference to established or generally agreed-upon representation, rules, or guidelines that the student is not following due to error or misconception.	<ul style="list-style-type: none"> When a student incorrectly orders fractions thinking the greater the denominator, the greater the fraction, the teacher might say, "Draw a picture of each fraction. What do you notice about the size of the fraction and the number in the denominator?" When a student is unsure about how to start solving a problem, the teacher says, "Think about which of the problem-solving strategies we have used might help you to get started."
Reflective	Promotion of metacognition—getting the student to think about his or her thinking—so that the student can use the resulting insight to determine next steps or the solution to a problem.	<ul style="list-style-type: none"> The student has just produced a solution incorrectly, and the teacher says, "Does that make sense? Think about the numbers you are working with and the meaning of the operation." A teacher says, "I see you're thinking strategically. What would be the next logical step?"
Heuristic	Engagement in an informal, self-directed, problem-solving procedure; the approach the student comes up with does not have to be like anyone else's approach, but it does need to work.	<ul style="list-style-type: none"> When the student does not get the correct answer to a math problem, the teacher says, "Maybe drawing a visual representation would help you see the problem." A teacher says, "Do you think you might find it easier to begin with a simpler but similar problem? What might that problem look like?"

Source: Adapted from Fisher and Frey (2014).

Figure 3.5

TYPES OF CUES FOR MATHEMATICS

Type of Cue	Definition	Example
Visual	A range of graphic hints to guide students' thinking or understanding	<ul style="list-style-type: none"> ● Highlighting areas within text where students have made errors ● Creating a graphic organizer to arrange content visually ● Asking students to take a second look at a graphic or visual from a textbook
Verbal	Variations in speech to draw attention to something specific or verbal "attention getters" that focus students' thinking	<ul style="list-style-type: none"> ● "This is important . . ." ● "This is the tricky part. Be careful and be sure to . . ." ● Repeating a student's statement using a questioning intonation ● Changing voice volume or speed for emphasis
Gestural	Body movements or motions to draw attention to something that has been missed	<ul style="list-style-type: none"> ● Making a predetermined hand motion such as equal or increasing ● Placing thumbs around a key idea in a problem that the student is missing
Environmental	Use of the classroom surroundings or physical objects in the environment to influence students' understanding	<ul style="list-style-type: none"> ● Using algebra tiles or other manipulatives ● Moving an object or person so that the orientation changes or the perspective is altered

Source: Adapted from Fisher and Frey (2014).

Figure 3.6



Video 3.3 Student Discourse That Builds Understanding

[http://resources.corwin.com/
VL-mathematics](http://resources.corwin.com/VL-mathematics)

answer make sense? Really think about it." can be used even following a correct answer to help that learner think about how to justify his or her thinking.

Too often, we ask questions or give students prompts or cues only when they are incorrect. Try asking students a question or providing a prompt when they are correct. Notice if they automatically assume they are incorrect because you stopped to ask a question. Productive questions, prompts, or cues should be a regular part of our instruction moves repertoire!

Conclusion

The tasks and assignments teachers provide for students are an important consideration. The wrong task may not only be a waste of time; it may fail to develop the type of thinking students need to be successful in mathematics. Quality tasks can be used to guide students' learning at the surface, deep, and transfer levels. These tasks can be considered across a number of dimensions, including difficulty and complexity. Assigning students ten more math problems may or may not ensure that they are engaged on complex thinking. In addition to the difficulty and complexity consideration, teachers have to consider the level of cognitive demand expected in the tasks and assignments they use to facilitate (and assess) learning.

We focused intentionally on the tasks and talk students must do before discussing types of learning. As we will see in the chapters that follow, understanding surface, deep, and transfer is really important, and identifying the right approach at the right time is a critical consideration for mathematics teachers and a key message from this book. Having said that, the type of learning students will do is based in large part on the types of tasks that teachers use during lessons. Again, a misalignment between the tasks and the types of learning puts students at risk. With the information about tasks in hand, we'll now turn our attention to the types of learning expected of students.

Reflection and Discussion Questions

1. Make notes of the questions you typically ask in your math lessons. Think about them in terms of the focusing and funneling questions framework discussed in this chapter. Which way does your questioning sequence lean? How can you make focusing questions a stronger presence in your mathematics classroom?
2. Identify two or three mathematics tasks you've asked your students to work on recently. Think about each task in light of its difficulty and complexity (see Figure 3.1). In which quadrant does each task fit? Is each the right kind of task given your learning intentions?
3. Think about these same tasks in terms of the cognitive demand they make on students (see Figure 3.2). How could you revise or reframe the tasks to require a higher level of cognitive demand?