Chapter 13

Independent-Samples *t* Test and Mann-Whitney *U* Test

13.1 Introduction and Objectives

This chapter continues the theme of hypothesis testing as an inferential statistical procedure. In the previous chapter, we investigated whether there was a significant difference between the mean of one random sample of a population and some hypothesized mean for that population.

We now address the situation where we compare two sample means. This test is known as the *independent-samples t test*, and its purpose is to see if there is statistical evidence that the two population means are significantly different. You may recall that the sample means are estimates of the unknown means in the sampled population. Therefore, if a significant difference is detected in the sample means, we make an inference that the unknown means of the population are also different. In the following section, you are given details regarding the independent nature of the samples and how the samples may be obtained for the independent-samples *t* test. You are also given the data assumptions required to use this test.

The *Mann-Whitney U test* is the alternative nonparametric test that may be used when the data assumptions required of the independent-samples *t* test cannot be met. Rather than comparing means, which requires scale

data, it uses the ranks of the values. Using ranks only requires that the data be measured at the *ordinal* level. However, the ultimate purpose of the Mann-Whitney *U* test is the same as that of the independent-samples *t* test—to search for statistical evidence that the sampled populations are significantly different.

OBJECTIVES

After completing this chapter, you will be able to

- Describe the data assumptions appropriate for using the independent-samples *t* test
- Write the research question and null hypothesis for the independent-samples *t* test
- Input data for, conduct, and interpret the independent-samples *t* test using SPSS
- Describe circumstances appropriate for the use of the Mann-Whitney *U* test
- Conduct and interpret the Mann-Whitney *U* test using two different approaches

13.2 RESEARCH SCENARIO AND TEST SELECTION

The scenario involves an investigation meant to determine if two makes of automobiles obtain significantly different gas mileages. The dependent variable is the number of miles per gallon (mpg) for the Solarbird and the Ecohawk. Recall that the *dependent* variable is the variable that is subject to change as a result of the manipulation of the *independent* variable. In this example, the independent variable is the type of automobile, Solarbird or Ecohawk. The experiment will use 12 Solarbird and 12 Ecohawk automobiles, each driven over identical courses for 350 miles each. What would be the appropriate statistical test to determine if the Solarbird and the Ecohawk get significantly different average gas mileages (dependent variable)?

We know that miles per gallon is *scale* data, which is a requirement of the *t* test. We also understand that the test vehicles will be randomly selected from a wide range of dealerships in the western United States. Random selection is another requirement of the *t* test. Prior research has also shown that the values for miles per gallon follow a normal curve and that the variances are approximately equal. Given this information, we see that all data requirements for the *t test* have been met.

Only one question remains: Will the samples be independent? As the scenario is explained, there will be two samples taken from two independent populations of Solarbirds and Ecohawks. Thus, we will have two independent samples. Based on this information, we select the *independent-samples t test* for this investigation.

Before moving on to the next section, we address how samples in the independent-samples *t* test may be obtained. Often, the concern over how the two samples may be obtained is a source of confusion when attempting to select the appropriate *t* test.

In the scenario just presented, it is very clear that you have two populations; a random sample is obtained from each, and then the sample means are compared. However, the clarity presented in this exercise regarding the independence of the samples is not always the case. For example, another sampling method might require that you take two random samples from one population and then compare the means of the two samples. Yet another alternative would be to take one random sample and divide this sample into two groups, perhaps males and females, and compare the means of these two groups. Regardless of the sampling process, the major consideration for the independent-samples *t* test is that the measurements taken on the samples must be independent. Independence means that the measurement is taken on another individual or object (e.g., an automobile). This is in contrast with the hypothesis test known as the *paired-samples t test*, which is presented in Chapter 14. In the *paired-samples t-test*, the measurements are taken on the same individual or object but at different times and/or under different conditions.

13.3 RESEARCH QUESTION AND NULL HYPOTHESIS

Before reading the next sentence, it would be instructive for you to look away and attempt to visualize the researcher's question, or the reason for conducting the investigation on these two automobile models. The researcher's idea is that there are statistically significant differences in the average (mean) miles per gallon for the Solarbird and Ecohawk automobiles. For the purpose of the testing procedure, we refer to the researcher's idea as the alternative hypothesis, and we write it as H_A: $\mu_1 - \mu_2 \neq 0$. In plain language, this expression simply states that the difference between the population means (for *all* Solarbirds and Ecohawks) is not equal to zero. The alternative hypothesis agrees with the researcher's idea—that there are differences in average miles per gallon for the Solarbird and Ecohawk automobiles.

The null hypothesis states the opposite and is written as H_0: $\mu_1 - \mu_2 = 0$. Once again in plain language, the null hypothesis depicts the outcome that the difference between the average miles per gallon for the two populations of automobiles is equal to zero. Remember that if the null hypothesis is rejected, the researcher will have statistical evidence in support of the alternative hypothesis.

13.4 DATA INPUT, ANALYSIS, AND INTERPRETATION OF OUTPUT

We begin this section by entering the variable information and the miles per gallon data for the 24 vehicles participating in this investigation. Solarbird and Ecohawk miles per gallon data are presented in Figure 13.1.

Figure 13.1 Miles per Gallon Data for the Independent-Samples *t* Test

Solarbird	34.5	36.2	33.2	37.0	32.7	33.1	30.5	37.2	33.5	32.0	36.2	35.7
Ecohawk	38.5	39.2	33.2	39.0	36.7	35.1	38.7	36.3	33.5	34.9	36.8	37.7

Next, we set up the SPSS database and then let the SPSS program do the hard work of analysis. The new database will consist of two variables: one for the miles per gallon data for all 24 vehicles and the other a *grouping* variable. The *grouping* (SPSS's term) variable simply labels the miles per gallon data as coming from a Solarbird or Ecohawk.

- Start SPSS, and click **Cancel** in the *SPSS Statistics* opening window.
- Click **File**, select **New**, and click **Data**.
- Click **Variable View**, type *mpg* (this is the name of the first variable), and then type *miles per gallon* in the cell below the *Label* column (select 2 decimal places and the *scale* level of measurement for this variable).
- Remain in the Variable View screen, and type *make* (this is the name of the second variable); then type *make of car* in the cell below the *Label* column (for this variable, set decimals to zero, and specify the *nominal* level of measurement in the *Measure* column).
- Click the right side of the cell below *Values*.
- The *Values* window opens; then type *1* in the *Value* box and *Solarbird* in the *Label* box, and click **Add**. Type *2* in the *Value* box and *Ecohawk* in the *Label* box, click **Add**, and then click **OK**.

- Click **Data View**, and type in all the "mpg" data (see Figure 13.1), beginning with *34.50* and ending with *37.70* (you should have data for all 24 cars—24 rows of data—entered in the first column of the Data View screen). Click the first cell below the "make" variable, and type *1* in the first 12 rows and *2* in the next 12 rows. (Now would be a good time to visually check the accuracy of your data entry.)
- Click **File**, then click **Save As** (the *Save Data As* window opens) in the *File name* box of the *Save Data As* window; type *miles_per_gallon* (note the underscores in the file name).
- Click **Save**.

You have now entered and saved the *miles_per_gallon* data for all vehicles. The data entry method just described, where you have all values for the dependent variable (miles per gallon) in one column, is required before SPSS will perform the test. Some may think that it would be more logical to enter two variables, one for Solarbird's miles per gallon and one for Ecohawk's miles per gallon, but SPSS does not work that way. Now let's do the fun part—the analysis—and see if we can discover the answer to the research question.

- Click **Analyze**, select **Compare Means**, and then click **Independent-Samples T Test**.
- Click **miles per gallon**, and then click the upper arrow (this moves the test variable to the right panel).
- Click **make of car**, and then click the lower arrow (this moves the *Grouping Variable* to the right panel, and the window should look like Figure 13.2; you may notice the question marks in the *Grouping Variable* box—don't worry as these will go away once you define the groups in the next step).
- Click **Define Groups** (the *Define Groups* window opens; see Figure 13.3).
- Click the *Group 1* box, and type *1* (1 is the *value* for the Solarbird).
- Click the *Group 2* box, and type *2* (2 is the *value* for the Ecohawk). (Completing this and the prior bullet point will eliminate the two *Grouping Variable* question marks seen in Figure 13.2.)
- Click **Continue**, and then click **OK** (the Output Viewer opens; see Figures 13.4 and 13.5).

The Output Viewer opens, displaying the tables shown in Figures 13.4 and 13.5. Figure 13.4 summarizes the descriptive statistics (*mean*, *standard deviation*, and *standard error*) for the "miles per gallon" variable. At first glance, we see that the Solarbird attained an average of 34.3167 mpg,

Figure 13.2 The *Independent-Samples T Test* Window

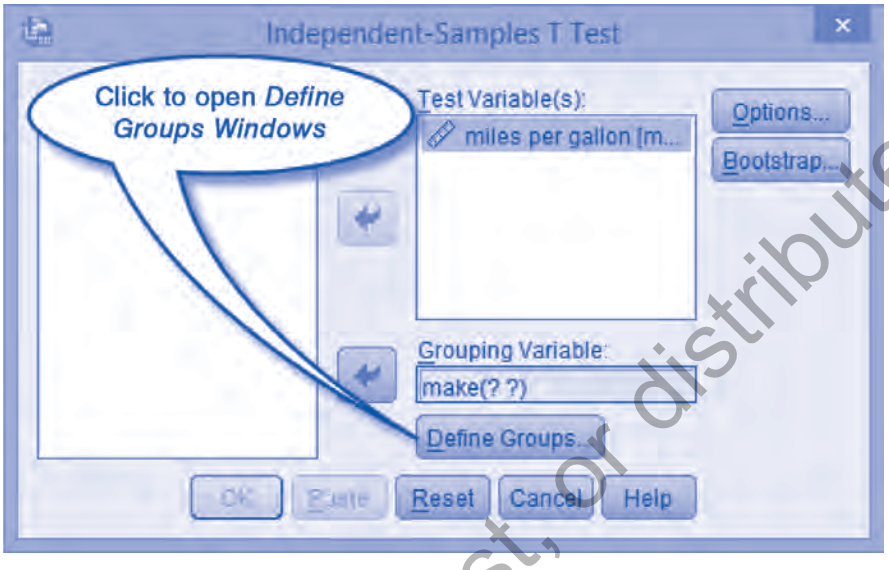

Figure 13.3 The *Define Groups* Window

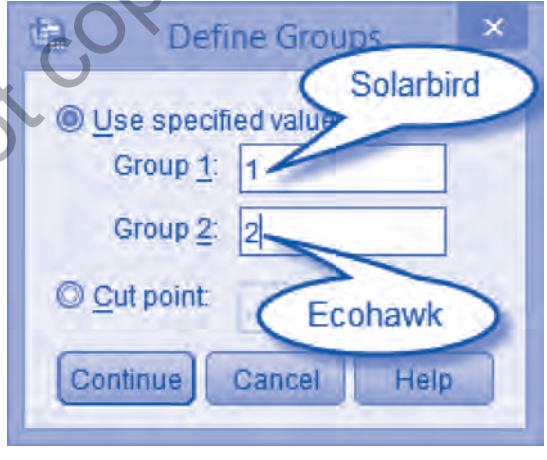

Figure 13.4 Group Statistics for Miles per Gallon

Group Statistics

	make of car	N	Mean	Std. Deviation	Std. Error Mean
miles per gallon	Solarbird	12	34.3167	2.14257	.61851
	Ecohawk	12	36.6333	2.09299	.60419

Figure 13.5 Independent-Samples *t* Test for Miles per Gallon

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means					
								95% Confidence Interval of the Difference	
					Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
miles per gallon	Equal variances assumed	.080	.780	-2.679	.014	-2.31667	.86484	-4.10982	-.52351
	Equal variances not assumed			-2.679	.014	-2.31667	.86484	-4.10988	-.52346

Large number (greater than .05) indicates equal variances

Small number (less than .05) then reject null hypothesis

whereas the Ecohawk managed an average of 36.6333 mpg. The standard deviations for average miles per gallon are close to being identical (2.14257 and 2.09299, respectively).

Figure 13.5 provides the answer to the research question. The researcher's idea is that there is a significant difference in the average miles per gallon for the Solarbird and Ecohawk automobiles. What does the *independent-samples t test* tell us?

When looking at Figure 13.5, we see that the 95% confidence interval of the difference (last two columns) ranges from −4.10982 to −.52351 when *equal variances are assumed*, which is the case for these data (Sig. = .780). Furthermore, we note that our calculated difference of −2.31667 is within this range of the confidence interval.

The significance level of .014 informs us that it is very unlikely that the observed mean absolute difference of 2.31667 was due to chance. Specifically, we can state that there was a .014 probability that the observed difference was the result of chance and that the null hypothesis can be rejected. Another way to state this is that there is only a 1.4% (100 × .014) chance that the differences in gas mileage could be attributed to chance. The difference can be taken seriously, and there are significant differences

in average miles per gallon attained by the Solarbird and Ecohawk. We further conclude that the Ecohawk gets superior gas mileage (36.63 mpg) when compared with the Solarbird's average of 34.32 mpg.

From the researcher's standpoint, we can say that the investigation was a success. The null hypothesis of no difference was rejected, and the researcher now has statistical evidence in support of the idea that these two makes of automobiles have significantly different rates of gas consumption.

13.5 NONPARAMETRIC TEST: MANN-WHITNEY *U* TEST

The miles per gallon data for the Solarbirds and Ecohawks were found to meet all the assumptions (scale data, equal variances, and normally distributed) required for the independent-samples *t* test. However, we wish to demonstrate the SPSS test when these assumptions are not met. We will use the same database for this demonstration. We expect that the less sensitive *Mann-Whitney U* test will also provide evidence that the gas mileage for the two vehicle makes will be significantly different.

The Mann-Whitney *U* test is the nonparametric test selected as the alternative to the independent-samples *t* test. The Mann-Whitney *U* test uses data measured at the *ordinal* level. Thus, SPSS *ranks* the miles per gallon scale data and then performs the statistical operations. Recall that it is a legitimate statistical manipulation to transform data from the higher levels of measurement (in this case *scale*) to lower levels (*ordinal*). The observations from both groups are combined and ranked, with the average rank assigned in the case of ties. If the populations are identical in location, then the ranks for miles per gallon should be randomly mixed between the two samples.

The *alternative hypothesis* is that the distributions of miles per gallon (ranks) are not equally distributed between the Solarbirds and Ecohawks. The *null hypothesis* is that the distributions of miles per gallon (ranks) are equal for both the Solarbirds and the Ecohawks.

Follow the bullet points to use the Mann-Whitney *U* test to discover if the null hypothesis can be rejected.

- Open SPSS, and open *miles_per_gallon.sav*.
- Click **Analyze**, select **Nonparametric Tests**, and then click **Independent Samples**.
- Click **Field**s in the *Nonparametric Tests: Two or More Independent Samples* window.
- Click **miles per gallon**, and then click the upper arrow.
- Click **make of car**, and then click the lower arrow (a window now appears as shown in Figure 13.6).

Figure 13.6 Nonparametric Tests: Two or More Independent Samples

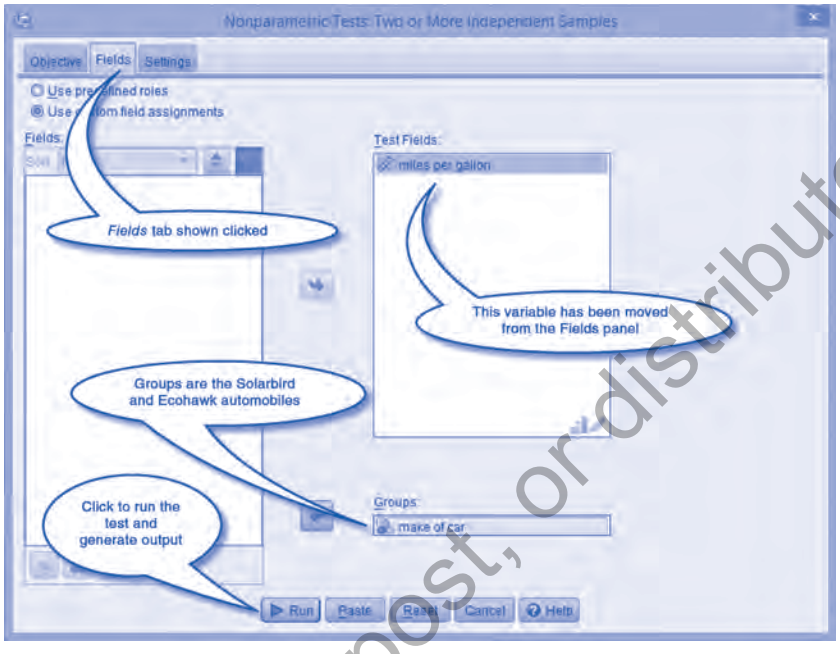

Figure 13.7 Hypothesis Test Summary for Miles per Gallon

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of miles per gallon is the same across categories of make of car.	Independent-Samples Mann-Whitney U Test	.014[1]	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

[1]Exact significance is displayed for this test.

- Click **Run** (the Output Viewer window opens; see Figure 13.7).

Figure 13.7 shows the SPSS output from the less sensitive, nonparametric Mann-Whitney *U* test. This test also found a significant difference (level of .014)

in the miles per gallon attained by these two automobiles. The null hypothesis of equality was rejected just as in the independent-samples *t* test.

13.6 SUMMARY

This chapter presented the parametric *independent-samples t test* and the nonparametric *Mann-Whitney U test*. Data were analyzed, which compared gas consumption (in miles per gallon) for two different makes of automobiles. The research was being conducted because the investigator suspected that the gas consumption would be different for the Solarbirds and Ecohawks. Data were given, and SPSS was used to generate inferential statistics for both parametric and nonparametric tests. Both indicated a significant difference, thus providing statistical evidence in support of the researchers' idea. In Chapter 14, the parametric *paired-samples t test* and its nonparametric analog, the *Wilcoxon signed-ranks* test, are presented and contrasted with the independent-samples tests presented here.

13.7 REVIEW EXERCISES

13.1 Two 12-man teams of Marines were randomly selected from Marine Corps Air Stations Miramar and Yuma to be compared on their Combat Fitness Test. Their scores ranged from a low of 263 to a perfect score of 300. Miramar scores: 267, 278, 295, 280, 268, 286, 300, 276, 278, 297, 298, and 279. Yuma scores: 263, 272, 286, 276, 267, 284, 293, 270, 272, 296, 279, and 274. The Yuma team leader and researcher had the idea that the scores were unequal. Can you help the Yuma team leader write the null and alternative hypotheses and select the appropriate test(s) to see if there is evidence in support his idea?

13.2 The local bank president had the idea that the money held in individual savings accounts would be significantly different for males and females. A random sample of the dollars in male and female savings accounts was recorded as follows. Males: 5,600, 5,468, 5,980, 7,890, 8,391, 9,350, 10,570, 12,600, 8,200, 7,680, 6,000, and 8,900. Females: 4,900, 5,200, 5,000, 7,000, 8,000, 9,050, 9,900, 12,000, 8,000, 7,500, 5,900, and 8,500. Write the null and alternative hypotheses, and select the correct test to seek evidence in support of the bank president's contention that male and female saving habits are significantly different.

13.3 For this review exercise, you will select and open the SPSS sample file called *bankloan.sav*. You will test for significant differences in the categories of education ("ed") and whether they have previously defaulted on a loan ("default"). There are five educational categories and two for the "default" variable. Write the alternative hypothesis and null hypothesis, and use the appropriate statistical test to see if the distribution of levels of education is the same for the categories that had previously defaulted.