CHAPTER 1

TEACHING MATHEMATICS

This chapter

- examines the typical motivations and anxieties of mathematics trainee teachers as they begin training
- summarizes the professional demands of training
- discusses different perceptions of the nature of mathematics; whether it is primarily a subject that is interesting in its own right, or whether it is primarily a tool for solving problems
- considers why people think that mathematics is important
- examines the extent to which mathematics is a human construct.

Your early days as a student teacher

Training to be a teacher is a challenging and demanding task, but one that provides enormous rewards. One of the biggest rewards is that moment when a pupil's eyes light up with a sense of understanding, and you know that a small piece of learning is down to you. During your training, you will feel the satisfaction of knowing that your professional skills are developing, and will have the opportunity to work alongside teams of dedicated staff, whose energy you will come to admire.

It is common for people embarking on a teacher-training course to have a number of anxieties. One of these anxieties often concerns subject knowledge. Many people starting training have come straight from university, and many come after a break from study of several years. Whichever background you have, you are likely to be 'rusty' on some elements of school mathematics, either because of the passage of time, or because the mathematical topics you studied at university were very abstract or specialized and did not require the use of topics from a school curriculum.

On your first day on the course, you will be relieved to find others like you: maybe other mature students, others whose first degree is not mathematics or others who may never have studied any mechanics. People come from a wide range of backgrounds wanting to become mathematics teachers.

During these early days, you may have some other anxieties: whether you will be able to control the class; whether you will cope with the workload; a worry that you will be placed in a difficult school. Such worries are common, but they can be answered:

- You will get lots of support from other teachers.
- You will not be given the worst class in the school.
- You will learn techniques of class management.
- Workload will be heavy, but it is manageable.
- Trainees who have been in 'difficult' schools are often very positive about their experiences afterwards.

It is important in the early days to get to know other students on the course; they will provide invaluable support as you progress through your training. Not only will they share with you their highs and lows, they will also be able to share ideas and resources that can save you time and effort.

As soon as the course begins, you will be asked to focus on the standards by which you will be judged during, and at the end of, your training. These standards are set nationally and are known as the Teachers' Standards (DfE, 2012, a link to which can be found on the companion website: www.sagepub.co.uk/chamberstimlin). The Standards document is divided into three parts: the Preamble, which explains the purpose of the standards; Part One which consists of eight standards for teaching with further amplification of the standard; and Part Two which outlines the standards for personal and professional conduct. In order to qualify as a teacher you will have to provide evidence that your practice is consistent with the principles outlined in the Preamble and that you meet each of the standards listed in Part One. A listing of the Standards is given in the 'How to use this book' section of this book, and a link to an electronic listing is provided on the website that accompanies the book. Your training course is designed to help you meet the standards.

One of the best ways to feel your way into the course is to develop your knowledge of the school mathematics curriculum. Whether directed to or not, it is helpful to get hold of a recent GCSE paper and work through it. The best exercise is for you to work through it as part of a group, discussing the bits that you have forgotten and explaining to others the bits that you can remember. It will not take long for your confidence to return, but make sure you check that the answers that you think are right are indeed the correct ones! Sometimes your confidence may be misplaced, and it is better to discover and correct your own misunderstandings in supportive company than when you are in the classroom.

Professionalism

Undertaking a teacher-training course is different from other university study. Because of the professional nature of the course, high standards of attendance, punctuality and commitment are expected, both in university and in schools. Remember that you are a guest in your teaching practice school; the school has volunteered to take a mathematics trainee, it is not obliged to take one.

When you go into school, you need to be aware of the behaviour and standards expected from teachers, and you should exhibit those standards from day one. Although you may not feel like a teacher on your first day, you need to behave like one; the pupils will observe the way that you are and will be forming impressions of you! In addition to the pupils, the teaching staff will be forming impressions of you and, obviously, it is important that you give the right message through the way you work and behave. Here are a few key pointers to acceptable behaviour in school:

- Arrive in plenty of time each morning; be punctual to lessons and meetings.
- Always let the school know if you are going to be absent.
- Treat all members of the school staff with respect. You need to demonstrate that you can work well with others.
- Dress smartly. (Play safe on day one, and after that take your cue from other members of the department.)
- Treat the pupils with respect. Show concern for their learning and welfare, but do not become over-friendly (especially when you are new to the school). Begin to build positive relationships, where pupils see you as someone to be trusted, someone who is fair and someone who can help them to achieve well.
- Set a good example to the pupils. This extends beyond your appearance and behaviour, and includes your values and attitudes; you should demonstrate positive attitudes and encourage the same in your pupils.

- Remain in school during the whole of the normal school day. In general, you should not leave the premises except at lunchtime.
- Use productively any non-contact time that you have. Do not be seen wasting time in the staff room; teachers are busy people, and they will expect to see you working as hard as they are. Do not fall into the trap of believing that your working day ends when teaching ends.
- Give positive support to colleagues, especially within the classroom.
- Take care to learn the school rules. Once you are part of the school staff (even as a trainee) you join in the collective responsibility for the implementation of school policies. This professional responsibility supersedes your personal opinion, so you must promote, for example, policies on school uniform, even if you disagree with pupils having to wear a uniform.

You will be assigned a mentor within the mathematics department. Mentors are key people in your training; they provide support and advice, and they also make judgements on you. You need to demonstrate your professionalism to all staff, but particularly to your mentor, throughout your time in school. For example, you can show your commitment to team-working through offering any original work you have done to the rest of the department. (Departments often have facilities for electronic sharing of worksheets, presentations and links.)

Above all, you need to be aware that during your time in schools, it is not only your skills as a teacher that are being assessed, but also your professional attributes. Part Two of the Teachers' Standards document is devoted to personal and professional conduct. There are three main headings with some further subdivisions giving eight statements which define the expectations. Having positive attitudes yourself helps, but you need to go beyond this; you must make it clear to everyone (pupils, teachers and, on occasions, parents) in all that you say and do, that you have high expectations and are committed to helping pupils to achieve their very best.

Motivations

The motivations for wanting to teach are many and varied. One of the authors' survey into the reasons for starting a PGCE mathematics course produced some interesting results, with the most common reasons given as:

- to pass on my enthusiasm for the subject
- to make a positive difference
- to do something worthwhile
- it is something that I expect to enjoy/find rewarding (Chambers, 2007).

Many respondents mention their enjoyment of doing mathematics. Sometimes they have a fascination for mathematics, and a curiosity that they wish to share with others; in other cases their love of mathematics is based on little more than a personal experience of success in the subject. Examples of trainees' motivations are given in the following quotes:

> I want to turn children on to mathematics, in the same way that I am.
>
> I had one particular mathematics teacher who I really admired, and she made all the difference to me. I would like to be that inspiring teacher to others.
>
> I love mathematics and yet it has such a negative image. I want to help change that image.
>
> I want to show children that mathematics is a fascinating subject.

Many people give their reason for wanting to teach as a desire to do something worthwhile, to feel that their efforts can make a difference to the lives of young people. Sometimes this follows many years spent working in industry, where the main motivation of the workforce is to make money. In comparison with this, teaching is seen as contributing to the common good and to the benefit of society.

Others mention their desire to work with young people. Many cite an experience that has helped confirm their decision. This experience may involve one or more of the following:

- helping out a friend or colleague with their mathematics
- coaching a child in school mathematics (possibly in preparation for an examination)
- working with youth groups (such as scouts, guides, church youth groups)
- observation of school mathematics lessons.

In practice, many training courses expect applicants to have spent some days in a secondary school, observing mathematics lessons. There is little doubt that this gives the best insight into whether mathematics teaching is something that will provide a suitable career.

One question in the survey concerned the trainees' major worries as they started the PGCE course in September. The most common anxiety is 'keeping control of my classes', which is understandable, particularly in view of press stories that tend to emphasize the sensational and the negative aspects of what goes on in schools. More surprisingly, the next biggest worry is coping with the anticipated workload during the training year. This scored considerably more highly than worries about the expected workload of being a teacher in the long term (although this was still the third most widespread anxiety).

Surprisingly, more than a third of the respondents are uncertain at the outset of the course whether teaching is the right career path for them. They start the course expecting to be successful, but they have a niggling worry about whether it is all going to work out. Smaller numbers of respondents mention anxieties about their own subject knowledge, difficult travel problems and whether the school staff would be welcoming.

Sadly, reactions from other people tend to be heavily dominated by negative attitudes to mathematics. When telling other people that they intend to train as a teacher of mathematics, the most common responses are:

- You must be mad.
- I could never do maths.
- You must be clever.
- I used to hate maths.

This gives some indication, if any more were needed, of the widespread fear and/or dislike of mathematics among the adult population.

Point for reflection

Consider your own motivations for wanting to become a mathematics teacher, and whether they fit in with the most common reasons given above. You have no doubt already discussed you decision to become a mathematics teacher with your family, friends and possibly former work colleagues; what were their reactions? Reflect on why they may have the negative image that they have. Consider why so many adults feel comfortable with (even proud of) their limited ability in mathematics. Was anyone envious of your decision?

List three things that you think you will enjoy about teaching.
List any aspects that you think you might not enjoy?

As you continue to talk to people from outside teaching during your training, note how many of them give a negative response to your intended career.

What is mathematics?

Anyone thinking of taking up the career of mathematics teacher needs, at some time, to consider this rather big question. In some ways, it is possible for specialist mathematicians in universities and schools to neglect it. Specialist mathematicians are often consumed by a knowledge of, and enthusiasm for, the subject. They take for granted that they understand what mathematics is, and that mathematics as a subject is a valuable area of study. At higher levels of mathematics, the study becomes so specialized that there is often no need for the bigger picture. Even within schools, there are teachers who have a thorough knowledge of the elements of the subject, but whose appreciation of the subject as a whole is weak.

Many people have tried to describe what mathematics is. Most definitions use words like logical ideas, interconnected ideas, relationships, patterns; some include other aspects such as communication, or particular sub-sections like the appreciation of number. Many discussions of the nature of mathematics make distinctions between mathematics as a subject to study in its own right and a subject that is useful. Ernest (1991) characterizes the distinction as deciding which is more important:

- understanding that $5 \times 23 = (4 \times 23) + (1 \times 23)$, or
- understanding that finding the cost of 5 apples at 23p each involves calculating 5×23, and knowing a way of doing it.

This distinction is typical of the different outlooks of pure mathematicians (purist views) and applied mathematicians or engineers (utilitarian views).

The views of the purists can be summarized in the following terms: mathematics is

- objective facts
- a study of reason and logic
- a system of rigour, purity and beauty
- free from societal influences
- self-contained
- interconnected structures.

From the point of view of the extreme purist, applied mathematics is looked down on as being based more on skills than understanding. Applications are inferior to the set of structures that make up pure mathematics, and whether or not a branch of mathematics is useful is an irrelevance. From this point of view, mathematics is a higher-level intellectual exercise, an art form and an example of the creativity of the human mind. Words like aesthetics and elegance are important to the purist (Scopes, 1973).

The description of mathematics as 'what mathematicians do' is sometimes used, but this seems to avoid giving a straight answer. Similarly, to describe the aim of performing mathematical investigations as a means 'to cultivate the art of doing mathematics' (Gardiner, 1987) seems to be insufficient justification. Smith (2004: 11) refers to the value of learning mathematics as being something that 'disciplines the mind, develops logical and critical reasoning, and develops analytical and problem solving skills to a high degree', which is a more helpful articulation of the purist standpoint. 'It must not be imagined', writes Bell (1953: 2), 'that the sole function of mathematics is to serve science . . . mathematics has a light and wisdom of its own, and it will richly reward any human being to catch a glimpse of what mathematics means to itself'. Similarly, Pedoe (1958: 9) refers to mathematics as not only being of interest to the science student; the subject is also of interest to those with arts backgrounds, and contains 'much which is beautiful and should interest everyone'.

The mathematics curriculum has tended to reflect the spirit of the times. The 1960s was a time of free expression, experimentation and challenges to authority, and it was purist views of mathematics that drove the curriculum changes that took place in UK mathematics classrooms in those years. Topics such as set theory, number bases and matrices were studied for the first time in the 11–16 curriculum. Transformation geometry replaced traditional Euclidean geometry, and there was more emphasis on probability and statistics. The philosophy behind these changes was that pupils needed the opportunity to see the rich structures in mathematics, whereas the existing curriculum had too much focus on routines and techniques. Many mathematics teachers embraced the changes with enthusiasm, but found it difficult to transfer this enthusiasm to parents, who regarded 'new maths' with suspicion.

In the 1980s, this view of mathematics gradually became replaced by a much more utilitarian view of the subject. The spirit of the times was much more geared towards economic success, and this was reflected in mathematics classrooms. Certainly by the end of the decade, education in general was being heavily influenced by the perceived needs of industry. Thus, applications became the most important part of mathematics, and it was from this point of view that the National Curriculum was first developed. In the utilitarian view of mathematics, learning how to do mathematics can become more important than understanding the underlying principles. Thus mathematics is characterized as

- a tool for solving problems
- the underpinning of scientific and technological study
- providing ways to model real situations.

By no means all the pressure for school mathematics to move away from its purist influences comes from the political field. Many in the mathematical world share the view that mathematics should be presented as something that is useful, and this view was highly influential in the introductions of the General Certificate of Secondary Education (GCSE) and the National Curriculum. Hence mathematics began to be taught more through applications and contextual situations, with the aim of increasing pupil motivation through demonstrating relevance (for example, Burkhardt, 1981; Mason et al., 1982). According to this view, mathematics is less about knowing and more about doing. There is an acceptance that pupils should study the pure mathematical skills, but it is the applications that bring the subject to life. This philosophy of teaching mathematics is backed up by pointing to the wide range of applications, 'the social sciences, biology and medicine, management, and, it seems, every field of human endeavour' where mathematics makes a contribution (Burghes and Wood, 1984).

A comprehensive revision of the mathematics National Curriculum in 2007 promoted the idea that mathematics is, above all, useful. There was an acknowledgement that mathematics has developed as 'a means of solving problems and also for its own sake' (QCA, 2007: 139), but the importance of mathematics was affirmed mainly through its usefulness: in business and finance, science and engineering, and in public decision-making. Critics decried the preference for usefulness, where if a mathematical topic is not used in everyday life, then it should be omitted from the curriculum. Typical of this criticism is the view that modern curricula have 'replaced the joy of arithmetic with the utilitarianism of numeracy' (Lingard, 2000: 40). The Smith Report (Smith, 2004) opens with several paragraphs about the place of mathematics within the curriculum. But the spirit of the times is perhaps reflected in the fact that only the first paragraph is devoted to a discussion of mathematics for its own sake. There then follow seven further paragraphs devoted to the usefulness of mathematics in a variety of fields – for the knowledge economy, for science, technology and engineering, and for the workplace. Contrast this with the summary of definitions of mathematics given by Orton (1994: 11): 'an organised body of knowledge, an abstract system of ideas, a useful tool, a key to understanding the world, a way of thinking, a deductive system, an intellectual challenge, a language, the purest possible logic, an aesthetic experience, a creation of the human mind', where the utility of the subject is only a minor aspect.

In January 2011 the Coalition Government ordered a comprehensive review of the whole National Curriculum. The reasons given for the review were that the National Curriculum had become too prescriptive in its provisions, and that teachers need greater freedom to construct their own programmes of study. The new programmes of study for mathematics (as well as those for English, science and physical education – PE) will commence in September 2013.

The National Curriculum itself is now envisaged as a much slimmed down document which reflects only the consensus of that body of essential knowledge necessary for learners at every stage of their school careers:

> The new National Curriculum will therefore have a greater focus on subject content, outlining the essential knowledge and understanding that pupils should be expected to have to enable them to take their place as educated members of society. (Ofsted, 2011: 1)

The utilitarian view of mathematics may remain the main influence on the curriculum, but it is still widely acknowledged that mathematics should be presented as a subject in its own right, a subject that can inspire and challenge at all levels. Two of the quotes chosen to introduce the 1999 version of the mathematics National Curriculum make a clear effort to indicate that mathematics is both interesting in itself and useful in solving problems.

> Mathematics is the study of patterns abstracted from the world around us – so anything we learn in maths has literally thousands of applications, in arts, sciences, finance, health and leisure! (Ruth Lawrence, quoted in DfEE, 1999a: 15)

> Nobody has to worry that pure mathematics won't be used. Mathematics – even some of the most abstruse mathematics that we thought would never be used – is now used every time you use your credit card, every time you use your computer. (Andrew Wiles, quoted in DfEE, 1999a: 15)

In addition, it can be argued (for example, Wigner, 1967) that the very fact that mathematics is so useful in describing the physical world is itself bordering on the mysterious. If the whole universe is a mathematical structure, then we are likely to wonder why this is the case. Hence the usefulness of mathematics brings us back to a study of the nature of mathematics itself.

So mathematics is a study of patterns, relationships and rich interconnected ideas (the purist view). It is also a tool for solving problems in a wide range of contexts (the utilitarian view). There is a third common answer to the question of what mathematics is, which says that mathematics is a means of communication. Mathematical language is a wonderful way of communicating ideas, which works across international boundaries, and is not subject to individual interpretations of meaning. Adrian Smith describes mathematics as providing 'a powerful universal language and intellectual toolkit for abstraction, generalisation and synthesis' (Smith, 2004: 11). Using mathematics, we 'convey ideas to each other that words can't handle' (Alison Wolf, quoted in DfEE, 1999a: 15).

Mathematics as a language has many facets. Within the English language, as in others, mathematics uses its own specialist vocabulary which helps to communicate specific ideas in a precise and unambiguous way. Developing this mathematical vocabulary is necessary if pupils are to have access to learning higher levels of mathematics, where specialists will routinely use this vocabulary. Another facet of mathematics as a language is algebra. This is a truly international language and is something that helps to bind the international mathematical community together. The use of Arabic letters for unknowns is universal, even where spoken languages use different scripts. Similarly, symbolic conventions like powers, roots, integrals, and so on, are recognized by the international community, so teaching mathematics helps pupils to have access to this rich body of internationally communicated ideas.

Another dimension to the debate about the nature of mathematics is the extent to which mathematics is a body of knowledge as opposed to a way of working. For example, in a book about mathematical investigations, Gardiner (1987) presents a series of problems that are not in themselves important, but 'what is important is the way the problems are studied'. Many in the mathematics education community agree with Gardiner that much teaching overemphasizes the content elements of the subject to the detriment of developing mathematical processes. They argue that it is more important to learn, for example, the skill of working systematically than the meaning of rotational symmetry. One is a mathematical fact; the other is a process that is useful in doing a range of mathematics.

Why should mathematics be taught?

All teachers are under pressure to produce good examination results, which can cause some to feel that a good set of results is the main purpose of their teaching. Although clearly this aspect of the job is high profile, it is important for you to consider the broader picture and examine why mathematics holds its position as part of the core curriculum.

If mathematics is mainly a tool for solving problems, then its reason for being in the curriculum is clear; it is so that pupils can acquire the skills they need to solve problems. If, on the other hand, mathematics is a fascinating body of knowledge or a means for appreciating patterns, then the reason for teaching it must be that it forms part of culture, and that an understanding of mathematics is required before anyone can be considered fully educated. This is clearly a more difficult idea to articulate, but is nevertheless a perfectly reasonable justification for teaching the subject.

The Mathematical Association identify a series of mathematical goals that define what mathematics teachers are trying to achieve (Mathematical Association, 1995: 8):

> The student should develop the ability to:
>
> - Read and understand a piece of mathematics
> - Communicate clearly and precisely using appropriate media
> - Work clearly and logically using appropriate language and notation
> - Use appropriate methods for manipulating numbers and symbols
> - Operate with shapes both in reality and in the imagination
> - Apply the sequence 'do, examine, predict, test, generalise, prove'
> - Construct and test mathematical models of real life situations
> - Analyse problems and select appropriate techniques for their solution
> - Use mathematical skills in everyday life
> - Use mechanical, technological and intellectual tools efficiently.

This list seems to include all the expected references to utilitarian aims, and makes clear reference to mathematical communication, but could be seen to under-represent the purist perspective. 'Work clearly and logically' is a mathematical skill concerned equally with thinking and with solving problems, and the concepts of generalizing and proving are key ideas from pure mathematics, but there is only one mention of the word 'understanding', and no explicit reference to appreciation of, or interest in, the subject.

The two bullet points on communication and the mathematical skills in everyday life are clearly important. On a basic level, mathematical language is part of everyday communication, and includes the huge number of graphic presentations used in the media to convey information. Hence it is important to teach mathematics so that pupils become informed citizens, who are able to understand information presented to them in a variety of graphical forms.

The importance of mathematics was stressed in the 2007 revision of the National Curriculum, which refers to mathematics as useful in the workplace and fundamental to national prosperity (QCA, 2007). But after a paragraph that is largely dominated by a utilitarian view of the subject, there is a short statement that mathematics is a creative discipline that 'can stimulate moments of pleasure and wonder for all pupils when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections' (QCA, 2007: 139).

Teaching mathematics is sometimes justified by the argument that it trains the mind (for example, QMUL, 2007; Smith, 2004), and is thus an aid to learning in other disciplines. As a justification in itself, this seems to overstate the case, and we need evidence of exactly what this training of the mind really means. It is more commonly accepted that acquiring general thinking skills is a cross-curricular aim of education rather than a justification for teaching any one particular subject.

Finally, in this section, we should mention the more general humanistic justifications for teaching mathematics. A study of mathematics contributes to societal values, how people feel about themselves and their environment (Bishop, 1991). Mathematics can provide people with a feeling of control over their environment, and therefore it increases a sense of power through knowledge. We are able to control events because we feel that they are predictable. Second, the study of mathematics suggests that problems can be solved, if not in full then in part. Mathematics thus reinforces the view that advances in society are possible and that aspirations to a better way of life are realistic. Third, mathematics reinforces a belief in rationalism. Things can be explained through logical argument; we can convince others of the correctness of our thinking through reason. Put together, these three justifications mean that mathematics helps us to feel more comfortable about the world where we live.

Point for reflection

Examine your own background to learning mathematics. In a utilitarian age, it is easy to justify mathematics in terms of its usefulness, but as specialist mathematicians, we should consider that the subject stands strongly in its own right. How would you convince someone that school mathematics should be studied because it is a worthwhile area of study that is part of human culture?

Numeracy and mathematics

'Numeracy' is one of those words whose meaning seems to have changed in recent years. Formerly, it was used to represent that subset of mathematics involving numbers, particularly understanding what numbers mean, and being able to perform calculations.

It then started to be used as shorthand for basic numeracy, understood as the sort of 'everyday' mathematics that all school leavers would need to cope with. This is the sense in which the word is used in the Cockcroft Report. In Cockcroft (1982), numeracy is being comfortable working with numbers, but also it is the set of mathematical skills used in daily life. This definition is itself open to interpretation, but most would concede that essential mathematics includes aspects of interpreting data, or using graphs, maps and scales that are not predominantly number work. It is in this sense that politicians and the media often use the word.

When, in 1996, the government set up a review into the teaching of mathematics in primary schools, the title chosen was the National Numeracy Project. Numeracy's connection with number work is retained in the working definition given by Askew et al. (1997), that numeracy is the ability to 'process, communicate, and interpret numerical information in a variety of contexts', but as the National Numeracy Project came to an end, its name was partly retained in the title of *The National Numeracy Strategy: Framework for Teaching Mathematics* (DfEE, 1999b). Hence in primary schools, the word 'numeracy' has become almost synonymous with the word 'mathematics'. Being numerate is understood as the ability to do mathematics (rather than any subset of mathematics).

There is some debate about whether there is still any useful distinction to be made in the use of the two words 'numeracy' and 'mathematics'. The discussion given by Tanner and Jones (2000) sees numeracy as a foundation for the whole of mathematics, rather than the whole thing. Numeracy involves 'an interaction between mathematical facts, mathematical processes, metacognitive self-knowledge, and affective aspects including self-confidence and the enjoyment *of number work*' (Tanner and Jones, 2000: 146, our italics). Our own view is that numeracy is virtually synonymous with mathematics, but with two differences. The first distinction is that numeracy is a slightly more active word than mathematics. Numeracy is less likely to be understood as a body of knowledge, and is more associated with doing mathematics. The second distinction is that numeracy has an (undefined) upper limit. Higher levels of study will always be called mathematics; lower levels of study may be called mathematics or numeracy.

The lack of a short verb to describe being numerate has caused difficulties for those wanting a headline or soundbite. In 1997, the Department for Education and Employment described the main functions of education as ensuring that every child can read, write and add up (DfEE, 1997). Mathematical basics have here been reduced to the very basic! In another example, when an employers' leader criticized the levels of school leavers' English and mathematics, one newspaper headline read, 'Bosses say school leavers can't read write or count' (Stewart, 2005). The rest of the article makes no reference to the ability to count, but refers instead to mathematics and problem-solving skills. The work that goes on in mathematics classrooms is thoroughly trivialized by this soundbite usage, where an ability with basic mathematics is equated to the ability to add up or to count.

Mathematics in the curriculum

The main reason why mathematics retains its place in the curriculum is that it is seen as useful, even though an analysis of the curriculum shows that very little of the content is used by most people in a normal week. Unfortunately, the emphasis on usefulness has not transferred to outcomes; although pupils may be able to perform mathematical operations in schools, they are often unable to transfer these skills to other social contexts (Lave, 1988). This inability to apply mathematics lies at the heart of the argument for greater emphasis on functional mathematics for all.

Functional skills are seen as the core elements of English, mathematics and information and communication technology (ICT) that enable individuals to access further education, be effective in the workplace and confident in their everyday life. Ofsted (2011) found that roughly 20 per cent of young adults enter the workforce without the necessary numeracy skills they need. The same report found that those aged 16–19 who were taking a vocational qualification which included a numeracy component were still being poorly served, with only half gaining a mathematics qualification at Level 2 or better. More effective teaching of mathematics in the secondary school focusing on the functional application of mathematics would certainly help remedy this situation.

Functional mathematics is more demanding than what in the past has been called basic skills or basic numeracy. To be successful in basic numeracy, candidates had to show that they could perform certain mathematical tasks that were seen as essential for everyday life, such as calculating percentages or interpreting graphs. Functional mathematics is concerned with those same skills, but also the ability to apply skills, to explain why and to justify conclusions. In other words, functional mathematics increases the importance of effectively communicating mathematical ideas. For example, tests of functional mathematics will examine candidates' ability to:

- use mathematics in different contexts
- draw conclusions and justify them
- interpret results and discuss their validity.

From 2013, all pupils will be required to continue in education if not in employment (in school/colleges or on apprenticeships) and one key question is what provision will be made for the teaching of mathematics to these learners. There have been calls for mathematics to remain compulsory to the age of 18, regardless of prior attainment (*Financial Times*, 2012). How schools and colleges might approach this is unclear. Providing for students who have not managed to achieve grade C at GCSE might require something more that 're-sit' classes, and there is no precedent for what those who do get grade C or better but choose not to take A level mathematics, might find both useful and stimulating in the further study of mathematics.

Evidence from the research

The philosophy of mathematics is a rich area for discussion. We have discussed the nature of the subject in terms of the purist/utilitarian viewpoints already, but there is also considerable debate about the extent to which mathematics is a social activity.

Traditional philosophers of mathematics treat it as a subject that stands on its own. It needs no input from other disciplines; it remains constant over time, and it is not affected by social constructs in any way. 'A theorem is true regardless of whether it is proven by a human, a computer or an alien' (Tegmark, 2003: 13). Others (for example, Hersh, 1998) argue the contrary: that mathematics must be understood as a human activity that has evolved historically, and which takes place in a social and cultural context. Such writers contend that there is a human dimension to the way in which mathematicians work.

This is one of the key discussions of mathematical philosophy. One group believes that mathematical truth is certain, that it is incontestable and entirely objective. For example, Kassem (2001: 72) reports 'a deep seated notion that the subject is value-free, independent of society and an exemplification of absolute truth'. This is known as the truth view of mathematics. Holders of this view are accused of an idealized view of the subject, ignoring how mathematics is, in favour of how it ought to be (Körner, 1960). The more commonly accepted view today is that mathematics is constructed; its truths are subject to argument, and may at any time in the future be challenged and revised. This is known as the constructivist view of mathematics.

All agree that all mathematical truths are proved from axioms, using the rules of inference. The constructivists argue that there is a fundamental fallacy in regarding mathematics as absolute truth. All mathematics uses deductive proof to demonstrate truths based on axiomatic starting points. But whatever axioms are chosen, they are simply chosen and not absolute. Examples of axioms may be that 1 + 1 = 2, or that the number of natural numbers is infinite. These may be thought to be above question, but their existence weakens the absolute truth view of mathematics. Ernest (1991: 13) argues that 'deductive logic only transmits truth, it does not inject it, and the conclusion of a logical proof is at best as certain as its weakest premise'.

Lakatos (1978) demonstrates the weakness of seeking certainty in mathematics. Any mathematical system depends on a set of assumptions. In order to prove an assumption we need to make earlier assumptions, and so on. We can never be free of the assumptions. The role of the mathematician is to reduce the number of assumptions to the smallest number possible.

> Mathematical truth ultimately depends on an irreducible set of assumptions, which are adopted without demonstration. But to qualify as true knowledge, the assumptions require a warrant for their assertion. But there is no valid warrant for mathematical knowledge other than demonstration and proof. Therefore the assumptions are beliefs, not knowledge and remain open to challenge, and thus to doubt. (Ernest, 1991: 14)

There is one further weakness in the truth view of mathematics: just as axioms are stated without proof, so the rules of deductive logic are themselves unprovable. Thus the foundations of mathematics as an unquestionable truth are weakened further.

So if we reject the philosophy of mathematics as truth, it becomes necessary to articulate a philosophy that mathematical truths are open to argument and can be refined over time. According to Hersh (1998), mathematical philosophy should not be about seeking universal truth. It should seek to give an account of mathematical knowledge as it really is: fallible, evolving and as subject to argument as every other branch of knowledge.

As discussed earlier in the chapter, mathematics is more than a body of knowledge; it is also an activity of gaining knowledge and understanding. As soon as we embrace this change in viewpoint, then mathematics becomes a human activity. Older philosophers saw mathematics as separate from other fields of human learning, but once it is accepted that mathematics is not infallible, the subject becomes part of the broader human knowledge that includes the sciences. According to this perspective, mathematics is part of society and hence a product of the culture that produced it. The development of mathematics is then subject to societal influences; it has values and cultural influences.

Putting a greater emphasis on the social side of mathematics has been propounded as a way of making the subject more interesting to more pupils. Lingard (2000) makes a strong case that learning about the history of mathematics may help motivation and hence achievement. The history of mathematics shows that the subject has developed over time (and is still developing) and reminds pupils that mathematicians are human.

If mathematics consists of a set of universal and incontrovertible truths, then it should not include apparent inconsistencies. At a simple level, it is possible to argue that the recurring decimal 0.99999 . . . is equal to one, and also that it is slightly less than one (see, for example, the argument on the nrich website – a link for which is provided on the website for this book at www.sagepub.co.uk/chamberstimlin). One famous challenge to the position of mathematics as a body of logical truths comes in the form of Russell's paradox. This suggests that we can separate out all sets in two piles, pile one for sets that are members of themselves and pile two for sets that are not members of themselves. If we then consider the set of all sets that are not members of themselves, then we have a paradox. We do not know which pile this set belongs in, because it appears to be a member of itself if and only if it is not a member of itself. The paradox illustrates the counter-intuitive fact: it is possible to find illogicalities in mathematics!

Point for reflection

Consider the extent to which mathematics is influenced by society. If mathematics is constructed rather than an absolute body of truth, then the social context of the time should influence how mathematics develops, and the sort of mathematical dialogue that goes on. Would mathematics develop in a totalitarian society in the same way as in a liberal democracy? Consider examples of how the mathematics would be independent of context, and examples where the contrary is the case.

Further reading

Orton, A. (1994) 'The aims of teaching mathematics', in A. Orton and G. Wain (eds), *Issues in Teaching Mathematics*. London: Cassell.

Orton's fairly short chapter is an interesting summary of the reasons for teaching mathematics, including discussion of the extent to which practice in schools is dominated by utilitarian aims. The author examines different ideas about the nature of mathematics, with particular emphasis on the distinction between learning a body of knowledge and learning mathematical processes. The chapter puts mathematics into the context of the aims of education as a whole, and also includes a section on mathematics as a language.

Ernest, P. (1991) *The Philosophy of Mathematics Education*. Basingstoke: Falmer Press.

The author is an acknowledged expert in the field of philosophy in mathematics education, and in this book he covers the big questions about the nature of mathematics and why we teach it. He compares different external influences on the mathematics curriculum, from both a purist and a utilitarian standpoint. He also considers the extent to which mathematics is a set of truths, and argues that mathematics should be regarded as a constructed entity, within a general philosophy of 'social constructivism'.

Useful websites

Live links to these sites can be found on the companion website.

The Teachers' Standards can be found on the website of the Department for Education (DfE), at http://www.education.gov.uk/schools/leadership/deployingstaff/a00205581/teachers-standards1-sep-2012-

A copy of the Smith Report on mathematics can be found online at http://www.mathsinquiry.org.uk/

The nrich website, run from Cambridge University, has discussion, articles, and enrichment problems that are linked in with the school curriculum. The home page is https://nrich. maths.org/. The page with the arguments on whether 0.9 recurring is equal to 1 or less than 1 is at https://nrich.maths.org/discus/messages/153904/68880.html?1143990391

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