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Observing, Studying, Analyzing, Planning

Preparing to Coach

Chapter Elements

- Case: Moving Between Models
- Math Activity: Models, Fractions, and Percents
- Focus Questions Activity

CASE DESCRIPTION

The Moving Between Models case offers an example of a reflective practitioner who studies the curriculum, the central mathematical idea in the classroom, the students' work as they engage in a new concept, and the implications for her coaching. Lila, the author, wrote the case midway into her second year as an elementary school coach. Specifically, she raises questions, both mathematical and pedagogical, about the relationship between fractions and percents and the issues that influence children's reasoning as they begin to work with ways to represent rational numbers.

The coach uses her case writing to help her learn more about the mathematics she encounters in the classroom and, at the same time, to help her analyze her next coaching moves and supports for the teacher.

The case offers an interesting perspective on the role of coaching. All coaching does not happen in the moment; certainly, there is a significant amount of preparation coaches undertake for individual classroom visits and meetings or workshops. The case raises a question for discussion: What is the nature of *preparing to coach?*

NOTES TO THE READER

As you read the case, write questions or comments in the margins and note the line numbers of sentences or paragraphs that interest you or raise questions for you.

Case 1 **Author: Lila**

Moving Between Models

In the past week, I've been in two fifth grade classrooms where the students are working through the ideas in the new curriculum unit. Both visits brought up a lot of questions for me about the ways students (and adults) think about fractions and percents and how they move between different models for these ideas.

Ms. Henry's class has had very little experience with this math curriculum prior to this year, and there is quite a range of understanding of fractions. She started off the math class by asking the students about the relationship between fractions and percents.

"Both talk about parts of a whole thing; it's just different ways to describe different parts of a whole thing," said Teddy.

"Both need to have equal parts that you cut them into and with percents there has to be 100 little parts," said Leonardo.

"We are learning about both of them," said Kayla.

Later, when we asked them to think about fractions that are the same as 60%, many said $\frac{60}{100}$. No one offered another fraction that represented the same amount. Many students could also write $\frac{27}{100}$ as 27%. I kept in mind that these are easily mimicked responses as the students began working on the day's story problems—ones that required more complex thinking and synthesis of ideas. The students' work with these problems was pretty widely variant and so interesting.

For one problem, students were asked to consider a class of 40 students, groups of which leave the classroom to do different things. One section of the problem says that 10 students left the class to go help first graders, and students are asked to calculate "what percent left the room?"

Ms. Henry's students' answers ran the gamut from $\frac{10}{10}$ to 10% to 25%. Similarly, although many students knew that if five students were absent that meant $\frac{5}{40}$ was absent, many said it meant that 5% were absent.

I became really intrigued. What accounts for the range of responses? And what about the equating of the numerator of a fraction with its percent equivalent, regardless of the denominator? What are these students' understandings and images for fractional parts and percents? How are their mental models of these ideas linked to their responses? What do they need help understanding in order to link, with more consistent success, the notions of fractions and percents?

All of this was still swirling around in my head a few days later when I spent some time in Mrs. Mittal's room at another school. The fifth grades in this school are pretty far behind in the pacing guide, so the session we saw was just the second activity. At this point in the unit, students calculate the shaded portion of a grid of one hundred squares and express that portion in terms of fractions and percents.

Because Mrs. Mittal was opening her classroom to several third and fourth grade teachers, she facilitated a previsit meeting for the visitors. Mrs. Mittal gave each of us a copy of her map of the session; she planned to start with some review warm-up material that included having students find factor pairs of 42.

In observing this class and, in particular, thinking about the work of Amari, one of Mrs. M's most vocal students, some ideas about the fractions and percents work kind of came together for me. So I want to begin by outlining Amari's work with the finding the factor pairs of 42 because it raised some interesting questions for me later on in the class. The students were doing this work on their whiteboards that they wave in the air to signal that they're done. When it was time to share, the class listed most of the factors pairs, including (6, 7) which were then scribed on the front board. Mrs. Mittal called for any other pairs.

"3 and 14!" Amari cried.

"How did you know?" Mrs. Mittal asked.

"Because if you have, like 6 and 7 . . . like six groups of seven things, well, it makes sense that if I had half as many groups I've got to have twice as many things in them to equal the same amount!"

Amari was loving her idea. Other students were confused, and Amari explained again. One student expressed frustration in not understanding the idea Amari discovered and was applying, and Mrs. Mittal suggested the two get together over break time to talk about it.

We moved into the beginning of the fraction and percent activity, where the students play a sort of rigged up game of "Guess My Rule" and brainstorm ways to express how many of the six students in the front of the room fit Mrs. Mittal's "rule" (students with zippers). They listed the following on the board:

3 out of 6

3

6

50%

Amari stared at the list, pretty distressed. "That's wrong," she said. "It's 3%, no, 30% . . ." but her voice trailed off.

"Why do you say that?" her teacher asked.

"Because it's 3 out of 6, and that's 30%, I mean $\frac{3}{6}$ is half, so $\frac{3}{6}$ is 30 . . ." Her voice trailed off as her own thinking started to feel "off" to her.

"Go back," said Mrs. Mittal. "Think about what you just said."

"$\frac{3}{6}$ is half . . . I see. We said that $\frac{1}{2}$ is 50%." Amari smiled and sat back, satisfied.

Way more happened after this, but I started to think about the flow of Amari's thinking. She understood 3 out of 6 as half, and she knew that half was 50%. But what was her understanding of 50%? While she seemed to understand that half could also *be written* as 50%, this feels different from understanding that $\frac{3}{6}$ is *equal to* 50%. So what models of these ideas were happening in her mind? Were they linked or still too disparate for her to move easily from one to another?

It made me think a lot about the different models of fraction/fractional ideas they're working with and that are possible to think about when working with the idea of portions. When Teddy mentioned his thought that fractions and percents both deal with parts of a whole, his thinking seemed based on an area model of portions—he was cutting up a square. The students have also established an area model for percent—a grid of a hundred squares (at least Ms. Henry's have; Mrs. Mittal's will be). They've worked with the notion of a set as well (How many students out of six are wearing stripes?) and used fractions to represent that. And they link that kind of idea to percents, but not necessarily because they see that grid of one hundred squares as a collection or set. In other words, I think it takes some time to apply that square area model to the notion of a set. What does it take to really see the link between the two?

I'm wondering if there's a step there that takes a lot of processing for students who do not really believe that fractions can refer to portions of groups of whole objects as well as to a piece of one whole object. How do students make the transition between an area model of a square of one hundred things and the thought of each of those squares representing one or part of a collection or a chance or whatever? How do they move between the notion of a whole as a discrete object to be sliced and a set of unsliced objects? What's in place that lets them see that $\frac{3}{6}$ is 50% because it can be seen as part of a set as well as part of a whole? This becomes weirder when I think that this can feel as if we are either extrapolating the set of 6 to a mythical set of 100, or subdividing the 6 into smaller bits so that we have 100 bits.

This makes me think of numerical ideas and multiplication, where we are also asking students to make that link between an area model and a collection—that a row of 4 squares in a 4×6 array actually could represent the legs of an elephant and that one square thus is representing one leg.

The notion of multiplication and manipulating groups was so clear to Amari. What led her to track so easily *the groups/number in a group* idea but have trouble when considering a collection and a portion of it? How do these ideas relate to quantity? What is it that she understands about quantity and constancy in the multiplication that isn't in yet in place with the "portion out of 100" idea? What makes a student ready to see that?

How do students keep track of the nature of the whole and move between different ideas of them?

It seems important to consider whether the array model is one that Amari uses when working with multiplication as she starts this work with percents. If not, then how *does* she think about it? And if not, is Amari's muddlement knit to the fact that her models of number and numeracy and multiplication are different? Is it that bags-of-candies kind of model that many people use? Four bags with three candies each/three bags with four candies each? If Amari doesn't use the array model for these ideas, it makes sense to me that she doesn't immediately knit the ideas of fractions and percents together.

So for Amari, as well as for the students in Ms Henry's class, what are the implications for teaching if this is the case? What if the students haven't yet internalized/understood the idea of percents as a collection of 100, or if they are thinking of it in terms of a cut-up whole into a 100 but haven't yet thought about each square as representing one thing? Is all of it made more complicated by the seeming arbitrariness of considering percents only in terms of 100 after the relative freedom of considering any number of parts to a whole when they worked with fractions? How does it all link to the idea of quantity and counting and tracking? What and how will the teachers and I learn in the coming weeks about students' models and the implications for, or influence on, students' reasoning?

As I think about these questions, I realize that they'll inform my next moves as a coach as well. Because, right now, I am thinking that an essential step in building an understanding of fractions is for kids to have ample and explicit opportunity to make connections between different fraction contexts and models, to compare them to make sense of the relationships between them. How do elements of one model relate to elements of another? How do fractions arranged on a line relate to a set of fraction strips? How is 25% percent on the grid like ¼ on the number line? Like ¼ of 100 cookies? How are they different? Because it is in such acts of comparison, I believe, that generalization is possible; that is, I believe that comparing models and situations helps kids make sense of the essence of fractions even as they come to understand the different contexts for their use. And for teachers to best support their kids in understanding these ideas, they need to feel comfortable with them as well.

So, as a coach, I need to make sure the teachers I work with have ample opportunities to think about these ideas—in fraction *and* in whole number contexts. ("How is using the array like skip counting on the hundreds chart?") This means that I'll need to make strategic use of looking at student work sessions, perhaps asking for specific samples or even bringing in work from other grade levels that scaffold and support such thinking. I also would love to facilitate teacher visits such as the one to Ms. Mittal's room, where the kids' thinking gives us so much fodder for thought. I can also envision using grade-level meetings to work through activities in the math unit and make connections between them, as well as math activities designed specifically for adult learners that invite teachers to make sense of the ideas.

But I also know that it is so important for me to hold a longer view of this work in mind; it's only our first year working together with this curriculum, and new understanding and ensuing shifts in pedagogy take time to take hold. But isn't this process what it's all about, for our students and for ourselves? All of us are learning.

Isn't that the best part of this work?

Case 1 **Moving Between Models**

Math Activity: Models, Fractions, and Percents

Here are a few math problems that help illustrate ideas in this case. Think carefully about your responses to these problems, note ideas you are relying on to solve them and draw models or representations on paper. The idea is to pay close attention to the mental images you construct that help you to solve these problems.

1. Jeffrey is showing 8 horses at the horse show in Northampton. He will have 5 of them in a jumping competition. What models might describe the fraction that represents the number of Jeffrey's horses in the jumping competition?
2. What percent of Jeffrey's horses are in the jumping competition? Describe a model that reflects that percent answer.
3. Jeffrey will bring a bag of oats to feed 3 horses at the show; the others eat only hay. One bag of oats is enough to feed 16 horses.
   a. For Jeffrey's 3 horses, how much of the bag will he use up?
   b. What are your ideas about modeling this fractional amount?
   c. Considering the same expressed as a percent, what model represents the amount?

Case 1 **Moving Between Models**

Focus Questions Activity

1. In lines 10–28 at the beginning of class, the coach describes a variety of student responses to translating fractions to percents. Consider these responses and discuss the logic of these early ideas.
2. Consider the student, Amari. To put Amari's struggles into context, the coach describes the flexibility of her thinking about multiplicative relationships. Refer to lines 49–60, and in your own words, describe Amari's mathematical idea. How general is the idea she states?
3. Amari appears confused about whether $\frac{3}{6}$ is equal to 50% or 3% or 30%. What is your interpretation of the coach's analysis?
4. Consider in your own words the questions the coach is contemplating in lines 102–134. What ideas about fractions and percents do students need to have in place to "move between models"?
5. What questions and insights does this case raise as you think about Amari's sorting out of numerical relationships and future work for Amari and her classmates?
6. Consider the coaching implications for Lila and these questions about your own coaching context:
   - What mathematical ideas and what pedagogical ideas do you think are important for teachers to work on?
   - What structures or opportunities in your own setting might allow for coaching with regard to these ideas?
   - What resources might you use for planning your coaching work with this teacher or with groups of teachers?