1

Mathematics Assessment Probes

To differentiate instruction effectively, teachers need diagnostic assessment strategies to gauge their students' prior knowledge and uncover their misunderstandings. By accurately identifying and addressing misunderstandings, teachers prevent their students from becoming frustrated and disenchanted with mathematics, which can reinforce the student preconception that "some people don't have the ability to do math." Diagnostic strategies also allow for instruction that builds on individual students' existing understandings while addressing their identified difficulties. The Mathematics Assessment Probes in this book allow teachers to target specific areas of difficulty as identified in research on student learning. Targeting specific areas of difficulty—for example, the transition from reasoning about whole numbers to understanding numbers that are expressed in relationship to other numbers (decimals and fractions)—focuses diagnostic assessment effectively (National Research Council, 2005, p. 310).

Mathematics Assessment Probes represent one approach to diagnostic assessment. They typically include a prompt or question and a series of responses. The probes specifically elicit prior understandings and commonly held misconceptions that may or may not have been uncovered during an instructional unit. This elicitation allows teachers to make instructional choices based on the specific needs of students. Examples of commonly held misconceptions elicited by a Mathematics Assessment Probe include ideas such as "multiplication makes bigger" and "the larger the denominator, the larger the fraction."

It is important to make the distinction between what we might call a silly mistake and a more fundamental one, which may be the product of a deep-rooted misunderstanding. It is not uncommon for different students to display the same misunderstanding every year. Being aware of and eliciting common misunderstandings and drawing students' attention to them can be a valuable teaching technique (Griffin & Madgwick, 2005).

The process of diagnosing student understandings and misunderstandings and making instructional decisions based on that information is the key to increasing students' mathematical knowledge. To use the Mathematics Assessment Probes for this purpose, teachers need to

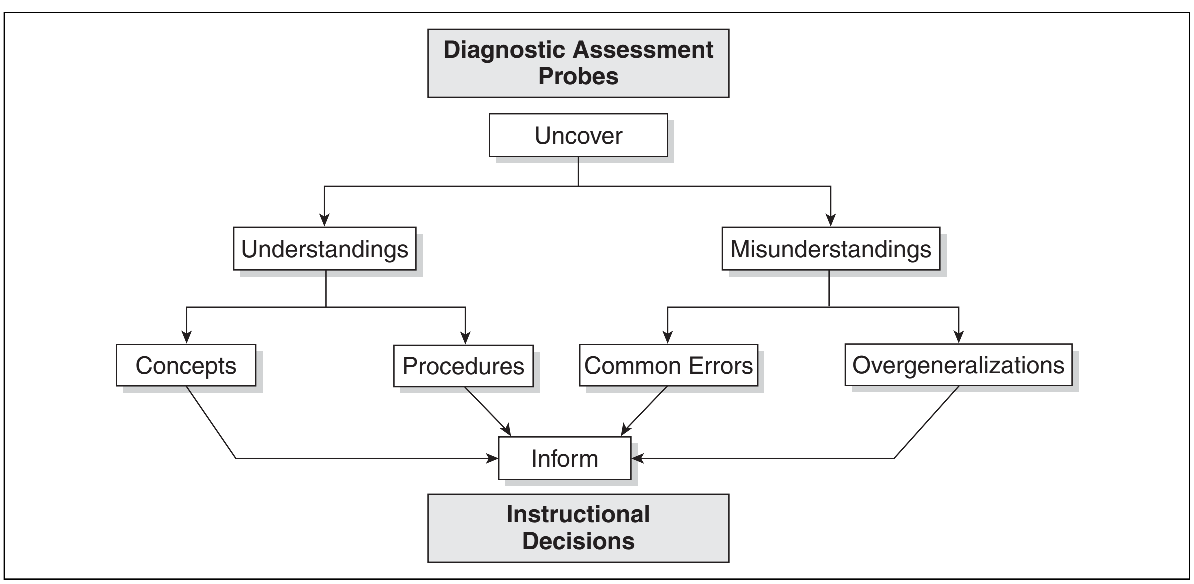
- Determine a question
- Use a probe to examine student understandings and misunderstandings
- Use links to cognitive research to drive next steps in instruction
- Implement the instructional unit or activity
- Determine the impact on learning by asking an additional question

The above process is described in detail in this chapter. The Teachers' Notes that accompany each of the Mathematics Assessment Probes in Chapters 3 through 5 include information on research findings and instructional implications relevant to the individual probe.

WHAT TYPES OF UNDERSTANDINGS AND MISUNDERSTANDINGS DOES A MATHEMATICS ASSESSMENT PROBE UNCOVER?

Developing understanding in mathematics is an important but difficult goal. Being aware of student difficulties and the sources of those difficulties, and designing instruction to diminish them, are important steps in achieving this goal (Yetkin, 2003). The Mathematics Assessment Probes are designed to uncover student understandings and misunderstandings; the results are used to inform instruction rather than make evaluative decisions. As shown in Figure 1.1, the understandings include both conceptual and procedural knowledge, and misunderstandings are classified as common errors or overgeneralizations. Each of these is described in more detail below.

Figure 1.1 Diagnostic Assessment Probes



Understandings: Conceptual and Procedural Knowledge

> Research has solidly established the importance of conceptual understanding in becoming proficient in a subject. When students understand mathematics, they are able to use their knowledge flexibly. They combine factual knowledge, procedural facility, and conceptual understanding in powerful ways. (National Council of Teachers of Mathematics [NCTM], 2000)

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they

- Recognize, label, and generate examples and non-examples of concepts
- Use and interrelate models, diagrams, manipulatives, and so on
- Know and apply facts and definitions
- Compare, contrast, and integrate concepts and principles
- Recognize, interpret, and apply signs, symbols, and terms
- Interpret assumptions and relationships in mathematical settings

Procedural Knowledge

Students demonstrate procedural knowledge in mathematics when they

- Select and apply appropriate procedures
- Verify or justify a procedure using concrete models or symbolic methods
- Extend or modify procedures to deal with factors in problem settings
- Use numerical algorithms
- Read and produce graphs and tables
- Execute geometric constructions
- Perform noncomputational skills such as rounding and ordering

(U.S. Department of Education, 2003, Chap. 4)

The relationship between understanding concepts and being proficient with procedures is complex. The following description gives an example of how the Mathematics Assessment Probes elicit conceptual or procedural understanding. The "Value of the Digit" probe (see Figure 1.2) is designed to elicit whether students understand place value beyond being able to procedurally connect numbers to their appropriate places. Students who are able to correctly choose (B) there is a 2 in the ones place and (E) there is a 1 in the tenths place, but do not choose (C) there are 21.3 tenths and (H) there are 213 hundredths, are often lacking a conceptual understanding of place value.

The following student responses to the "Explain Your Reasoning" prompt are indicative of conceptual understanding of place value:

> I know for a whole number like 253 the 2 means 200 and the 5 means 50 so for decimals it is similar but opposite because it is a part of a

whole number. For the 2.13 the three is in the 100th place which means there is $^{213}/_{100}$.

Expanded, the number 2.13 is $2 + {}^{1}/_{10} + {}^{3}/_{100}$. How many 10th? Look at $2 + {}^{1}/_{10}$ or .1 plus the extra $^{3}/_{100}$ or .03. How many 100th? Combine all three.

Figure 1.2 The Value of the Digit

The Value of the Digit
Circle all of the statements that are true for the number 2.13.

A. There is a 3 in the ones place.
B. There is a 2 in the ones place.
C. There are 21.3 tenths.
D. There are 13 tenths.
E. There is a 1 in the tenths place.
F. There is a 3 in the tenths place.
G. There are 21 hundredths.
H. There are 213 hundredths.

For more information see page 33

Misunderstandings: Common Errors and Overgeneralizations

In *Hispanic and Anglo Students' Misconceptions in Mathematics,* Jose Mestre summarizes cognitive research as follows: Students do not come to the classroom as "blank slates" (Resnick, 1983, quoted in Mestre, 1989). Instead, they come with theories constructed from their everyday experiences. They have actively constructed these theories, an activity crucial to all successful learning. Some of the theories that students use to make sense of the world are, however, incomplete half-truths (Mestre, 1989). They are misconceptions.

Misconceptions are a problem for two reasons. First, they interfere with learning when students use them to interpret new experiences. Second, students are emotionally and intellectually attached to their misconceptions because they have actively constructed them. Hence, students give up their misconceptions, which can have such a harmful effect on learning, only with great reluctance. For the purposes of this book, these misunderstandings or misconceptions will be categorized into common errors and overgeneralizations. Each of these categories of misunderstandings is described in more detail below.

Common Error Patterns

Common error patterns refer to systematic uses of inaccurate/inefficient procedures or strategies. Typically, this type of error pattern indicates non-understanding of an important math concept (University of Kansas, 2005). Examples of common error patterns include consistent misuse of a tool or steps of an algorithm, such as an inaccurate procedure for computing or the misreading of a measurement device. The following description gives an example of how the Mathematics Assessment Probes elicit common error patterns.

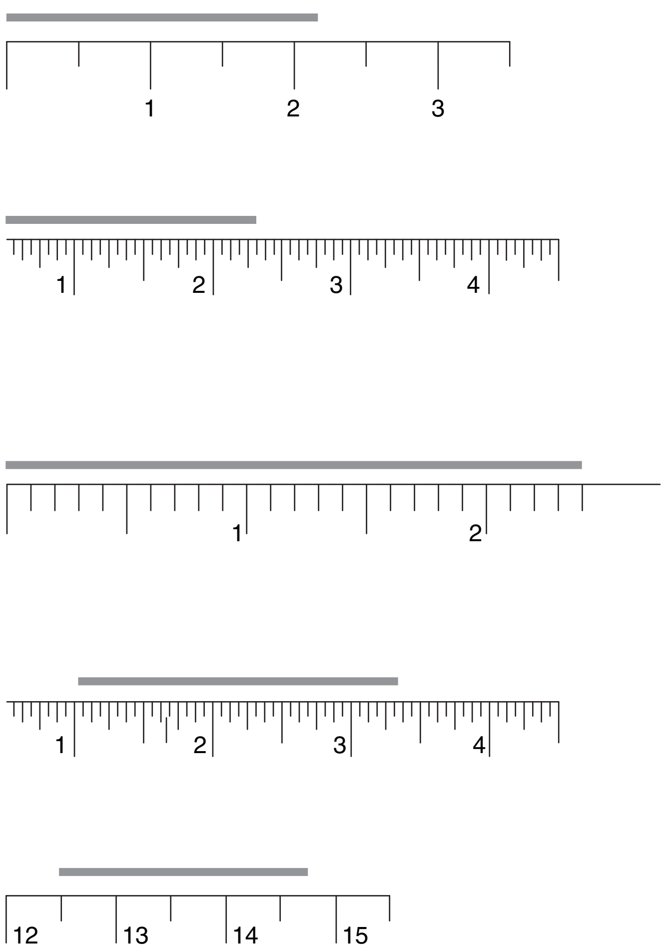
One of the ideas the "What's the Measure?" probe (see Figure 1.3) is designed to elicit is the understanding of zero point. "A significant minority of older children (e.g., fifth grade) respond to nonzero origins by simply reading off whatever number on a ruler aligns with the end of the object (Lehrer et al., 1998a)" (NCTM, 2003, p. 183). We have found many middle school students also make this same mistake.

The correct answers are A, C, D, and E. For students who incorrectly choose B and do not choose D and E, their error is typically based on the common error of not considering the beginning point on the ruler as it relates to the beginning of the object being measured.

Figure 1.3 What's the Measure?

What's the Measure? Name: ___________________________

Circle the letter beside each line segment that is approximately $2\frac{1}{4}$ units long, as measured by the standard or nonstandard tool provided below each of the segments.



A.

B.

C.

D.

E.

For more information see page 107

Overgeneralizations

Often, students learn an algorithm, rule, or shortcut and then extend this information to another context in an inappropriate way. These misunderstandings are often overgeneralizations from cases that students have seen in prior instruction (Griffin & Madgwick, 2005). To teach in a way that avoids creating any misconceptions is not possible, and we have to accept that students will make some incorrect generalizations that will remain hidden unless the teacher makes specific efforts to uncover them (Askew & Wiliam, 1995).

The following example illustrates how the Mathematics Assessment Probes can elicit overgeneralizations. The "Is It a Variable? Probe (see Figure 1.4) is designed to elicit the overgeneralization of the use of letters and symbols to represent variables. Students often overgeneralize from the general definition of a variable, a letter or symbol that represents a quantity, leading to identifying any letter or symbol used in a mathematical situation as a variable. Students who incorrectly choose B, C, and D often have overgeneralized in this way.

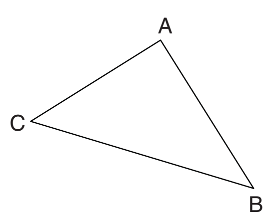
In addition to uncovering common misunderstandings, the Mathematics Assessment Probes also elicit uncommon misconceptions that may not be uncovered and could continue to cause difficulty in understanding a targeted concept. An example of this is highlighted in the following Image From Practice.

Figure 1.4 Is It a Variable?

Is It a Variable?
Circle the letter of each situation that contains a variable.

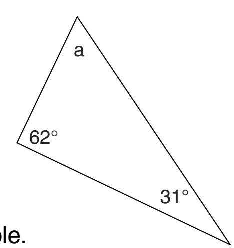
A. $\square + 2 = 8$

B. 2m = 200cm

C. 

D. $3 \times 2 + 2 = 8$

E. $n = \frac{100}{m}$

F. 

Explain why each letter that you circled contains a variable.

For more information see page 200

Image From Practice: Gumballs in a Jar

While reviewing student responses to Gumballs in a Jar [see Figure 1.6], I was intrigued by one particular student's response that didn't seem to fit any of the suggested typical misunderstandings. Having just spent time on using red and white chips to discover the rules of adding and subtracting integers, the student responded, "Jar B is better because if you take out zero pairs from Jar A [you leave] a chance of +1 and tak[ing] zero pairs from Jar B leav[es] a chance of +2." Although I was not expecting this misunderstanding, thinking about misconceptions as overgeneralized ideas helped me realize just how easy it is for students to make incorrect connections across processes and concepts.

HOW WERE THE MATHEMATICS ASSESSMENT PROBES DEVELOPED?

Developing an assessment probe is different from creating appropriate questions for summative quizzes, tests, or state and national exams. The probes in this book were developed using the process described in *Mathematics Curriculum Topic Study: Bridging the Gap Between Standards and Practice* (Keeley & Rose, 2007). The process is summarized as follows:

- Identify the topic you plan to teach and use national standards to examine concepts and specific ideas related to the topic. The national standards used to develop the probes for this book were NCTM's (2000) *Principles and Standards for School Mathematics* and the American Association for the Advancement of Science (AAAS)'s *Benchmarks for Science Literacy* (AAAS, 1993).

- Select the specific concepts or ideas you plan to address and identify the relevant research findings. The source for research findings include NCTM's *Research Companion to Principles and Standards for School Mathematics* (2003), Chapter 15 of AAAS's (1993) *Benchmarks for Science Literacy,* and additional supplemental articles related to a topic.

- Focus on a concept or a specific idea you plan to address with the probe, and identify the related research findings. Choose the type of probe format that lends itself to the situation (see more information on probe format following the Gumballs in a Jar example on p. 9). Develop the stem (the prompt), key (correct response), and distracters (incorrect responses derived from research findings) that match the developmental level of your students.

- Share your assessment probe(s) with colleagues for constructive feedback, pilot with students, and modify as needed.

Figure 1.5, which is taken from Keeley and Rose (2007), provides the list of concepts and specific ideas related to the probability of simple events.

The shaded information was used as the focus in the development of the probe, Gumballs in a Jar (see Figure 1.6). The probe is used to reveal common

Figure 1.5 Probability Example

Topic: Probability (Simple Events)

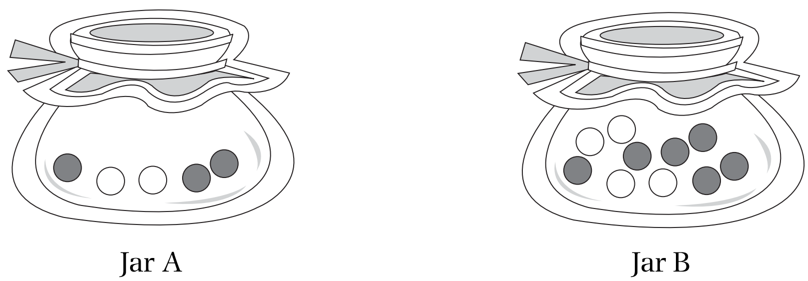
Concepts and Ideas	Research Findings
• Events can be described in terms of being more or less likely, impossible, or certain. (*BSL,* 3–5, p. 228) • Probability is the measure of the likelihood of an event and can be represented by a number from 0 to 1. (*PSSM,* 3–5, p. 176) • Understand that 0 represents the probability of an impossible event and 1 represents the probability of a certain event. (PSSM, 3–5, p. 181) • Probabilities are ratios and can be expressed as fractions, percentages, or odds. (*BSL*, 6–8, p. 229) • Methods such as organized lists, tree diagrams, and area models are helpful in finding the number of possible outcomes. (*PSSM,* 6–8, pp. 254–255) • The theoretical probability of a simple event can be found using the ratio of # favorable outcome/total possible outcomes. (*BSL,* 6–8, p. 229) • The probability of an outcome can be tested with simple experiments and simulations. (*PSSM,* 6–8, pp. 254–255) • The relative frequency (experimental probability) can be computed using data generated from an experiment or simulation. (*PSSM,* 6–8, pp. 254–255) • The experimental and theoretical probability of an event should be compared with discrepancies between predictions and outcomes from a large and representative sample taken seriously. (*PSSM,* 6–8, pp. 254–255)	**Understandings of Probability** (*Research Companion,* pp. 216–223) • Lack of understanding of ratio leads to difficulties in understanding of chance. • Students tend to focus on absolute rather than relative size. • Although young children do not have a complete understanding of ratio, they have some intuitions of chance and randomness. • A continuum of probabilistic thinking includes subjective, transitional, informal quantitative, and numerical levels. • Third grade (approx.) is an appropriate place to begin systematic instruction. • “Equiprobability” is the notion that all outcomes are equally likely, disregarding relative and absolute size. • The outcome approach is defined as the misconception of predicting the outcome of an experiment rather than what is likely to occur. Typical responses to questions are “anything can happen.” • Intuitive reasoning may lead to incorrect responses. Categories include representativeness and availability. • Wording of task may influence reasoning. • NAEP results show fourth and eighth graders have difficulty with tasks involving probability as a ratio of “m chances out of n” but not with “1 chance out of n.” • Increased understanding of sample space stems from multiple opportunities to determine and discuss possible outcomes and predict and test using simple experiments. **Uncertainty** (*BSL,* Chap. 15, p. 353) • Upper elementary students can give correct examples for certain, possible, and impossible events, but have difficulties calculating the probability of independent and dependent events. • Upper elementary students create “part to part” rather than “part to whole” relationships.

NOTES: BSL = *Benchmarks for Science Literacy*; PSSM = *Principles and Standards for School Mathematics*.

Figure 1.6 Gumballs

Gumballs in a Jar

Two jars have black and white gumballs.
Jar A: 3 black and 2 white
Jar B: 6 black and 4 white
Which response best describes the chance of getting a *black* gumball?



A. There is a better chance of getting a black gumball from Jar A.
B. There is a better chance of getting a black gumball from Jar B.
C. The chance of getting a black gumball is the same for both Jar A and Jar B.
Explain your reason(s) for the answer you selected.

errors regarding probability, such as focusing on absolute size, or a lack of conceptual understanding of probability as a prediction of what is likely to happen. There is the same chance you will pick a black gumball out of each jar. Jar A has a probability of $3/5$, and Jar B has a probability of $6/10 = 3/5$. There are a variety of trends in correct thinking related to this probe, some of which are doubling, ratios, and percents. Some students might correctly choose answer C but use incorrect reasoning such as "you can't know for sure since anything can happen," an explanation that indicates a lack of conceptual understanding of probability. Other students may demonstrate partial understanding with responses such as "each jar has more black than white." Some students reason there are fewer white gumballs in Jar A compared to Jar B and therefore there is a better chance of picking a black gumball from Jar A.

Others observe that Jar B has more black gumballs compared to Jar A and therefore reason that there is a better chance of picking a black gumball. In both cases, students are focusing on absolute size instead of relative size in comparing the likelihood of events. Students sometimes choose Distracter A due to an error in counting or calculation.

Additional probes can be written using the same list of concepts and specific ideas related to the probability of simple events. For example, by focusing on the statement from the research, "NAEP results show fourth and eighth graders have difficulty with tasks involving probability as a ratio of 'm chances out of n' but not with '1 chance out of n'" a probe using an example of each can diagnose if students are demonstrating this difficulty.

WHAT IS THE STRUCTURE OF A MATHEMATICS ASSESSMENT PROBE?

The probes are designed to include two tiers, one for elicitation of common understandings and misunderstandings and the other for the elaboration of individual student thinking. Each of the tiers is described in more detail below.

Tier 1: Elicitation

Since the elicitation tier is designed to uncover common understandings and misunderstandings, a structured format using stems, correct answers, and distracters is used to narrow ideas found in the related research. The formats typically fall into one of six categories.

Selected Response

One stem, one correct answer, and several distracters (see Figure 1.7)

Figure 1.7 What's Your Estimate?

What's Your Estimate?

Use **mental math** to answer the following question:

Circle the best estimate:

$\frac{12}{13}+\frac{7}{8}$

A. 1
B. 2
C. 19
D. 21

Explain your reasoning.

For more information see page 51

Multiple Selections Response

- Two or more sets of problems, each with one stem, one correct answer, and one or more distracters (see Figure 1.8)

Figure 1.8 Expressions: Equal or Not Equal?

Expressions: Equal or Not Equal?

A

πr^2 and $2\pi r$
Equal or not equal?
Explain:

B

$2\ (l{+}w)$ and $2l + w$
Equal or not equal?
Explain:
For more information see page 158

C

lwh and hwl
Equal or not equal?
Explain:

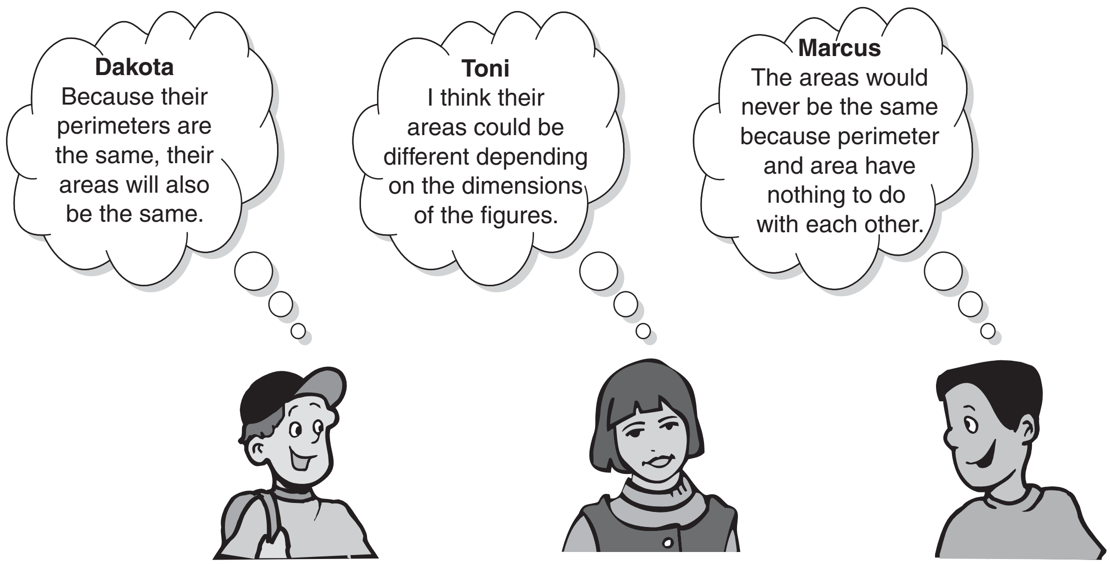
Opposing Views/Answers

- Two or more statements are provided and students are asked to choose the statement they agree with (see Figure 1.9). This format is adapted from *Concept Cartoons in Education,* created by Stuart Naylor and Brenda Keogh (2000) for probing student ideas in science.

Figure 1.9 Are Area and Perimeter Related?

Are Area and Perimeter Related?

Three classmates were having a discussion about two figures that have the same perimeter.



Which classmate do you agree with? ______________________

For more information see page 112

Explain why you agree with one classmate but disagree with the others.

Examples and Non-Examples List

- Several examples and non-examples are given, and students are asked to check only the examples based on a given statement (see Figure 1.10).

Justified List

- Two or more separate problems or statements with each answer choice needing to be justified (see Figure 1.11).

Figure 1.10 Is It True?

Is It True?

If m = 5, circle all of the statements below that are true for the expression 3m.

A. 3m = 35

B. 3m = 8

C. 3m = 3 + 5

D. 3m = 3 meters

E. 3m = 15

F. 3m = 3 times 5

G. 3m means the slope is $^3/_5$

H. 3m = 3 miles

For more information see page 180

Explain your reasoning for each circled statement.

Figure 1.11 What Do You *Mean?*

What Do You *Mean*?

Each statement below can be preceded by one of the following statements:

The mean is always . . .

The mean is sometimes . . .

The mean is never . . .

Read each statement and indicate **A** (always), **S** (sometimes), or **N** (never):

For more information see page 136

Statement	Justify Response
1. ☐ the value obtained by dividing the sum of a set of data points by the number of data points in the set.	
2. ☐ equal to the value of the term in the middle.	
3. ☐ equivalent to the value of the mode.	
4. ☐ changed when the same amount is added to each of the data points.	
5. ☐ affected when a 0 is added as one of the data points.	
6. ☐ one of the data points in the original set.	

Strategy Elicitation

- A problem is stated with multiple solution strategies given. Students provide an explanation regarding making sense of each strategy. (see Figure 1.12).

Figure 1.12 What's Your Addition Strategy? Decimals

Sam, Julie, Pete, and Lisa each added the numbers 11.5 + 2.7.

Sam's Method: "I broke the 2.7 apart." 11.5 + 2.7 11.5 + 2 = 13.5 13.5 + .5 = 14 14 + .2 = 14.2	**Does each of the methods make sense mathematically? Why or why not?**
Pete's Method: "I rounded up, then subtracted the extra." 11.5 + 2.7 11.5 + 3 = 14.5 14.5 − .3 = 14.2	For more information see page 82
Julie's Method: "I added the numbers in the columns." $\overset{1}{1}1.5$ 2.7 ——— 14.2	

Tier 2: Elaboration

The second tier of each of the probes is designed for individual elaboration of the reasoning used to respond to the question asked in the first tier. Mathematics teachers gain a wealth of information by delving into the thinking behind students' answers, not just when answers are wrong but also when they are correct (Burns, 2005). Although the Tier 1 answers and distracters are designed around common understandings and misunderstandings, the elaboration tier allows educators to look more deeply at student thinking. Often a student chooses a specific response, correct or incorrect, for an atypical reason. Also, there are many different ways to approach a problem correctly; therefore, the elaboration tier allows educators to look for trends in thinking and in methods used.

WHAT ADDITIONAL INFORMATION IS PROVIDED WITH EACH MATHEMATICS ASSESSMENT PROBE?

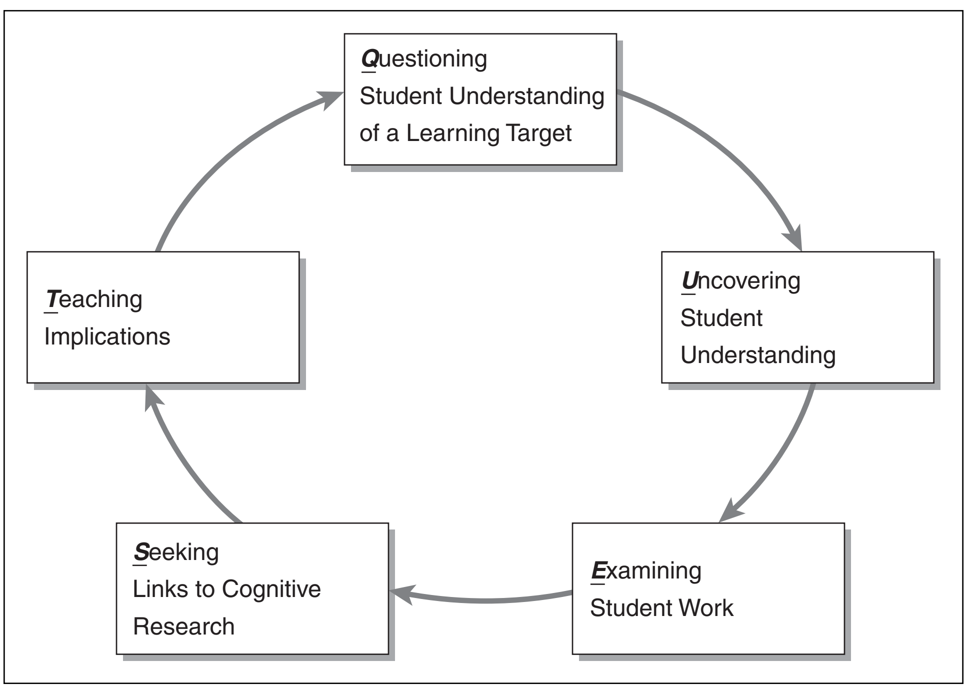
In *Designing Professional Development for Teachers of Science and Mathematics,* Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003) describe action research as an effective professional development strategy. To use the probes in this manner, it is important to consider the complete implementation process.

We refer to an action research *quest* as working through the full cycle of

- Questioning student understanding of a particular concept
- Uncovering understandings and misunderstandings using a probe
- Examining student work
- Seeking links to cognitive research to drive next steps in instruction
- Teaching implications based on findings and determining impact on learning by asking an additional question

The Teachers' Notes, included with each probe, have been designed around the action research QUEST cycle, and each set of notes includes relevant information for each component of the cycle (see Figure 1.13). These components are described in detail below.

Figure 1.13 QUEST Cycle



***Q**uestioning student understanding of a particular concept.* This component helps to focus a teacher on what a particular probe elicits and to provide information on grade-appropriate knowledge. Figure 1.14 shows an example question from the Mathematics Assessment Probe, What's the Capacity?

Figure 1.14 Questioning Student Understanding of a Particular Concept

Example Question*: What's the Capacity?*

***Question**: Do students recognize how a change in the dimensions affects the volume of a figure?*

Grade Level:

6–8	9–12

Grade span bars are provided to indicate the developmentally appropriate level of mathematics as aligned to the NCTM standards and cognitive research. The dark gray band represents the grade levels where the mathematics required of the probe is aligned to the standards, and the lighter gray band shows where field-testing has indicated students still have difficulties.

Uncovering understanding by giving the Mathematics Assessment Probe to students. Figure 1.15 shows an example, Uncovering Understanding, from the Mathematics Assessment Probe, What's the Capacity?

Figure 1.15 Uncovering Understandings

Example: *What's the Capacity?*

Uncover Understandings Using the Following Mathematics Assessment Probe:

What's the Capacity? (Content Standard: Geometry and Measurement)

***E**xamining student work.* This section includes information about the stem, answer(s), and distracters as related to the research on cognitive learning. Example student responses are given for a selected number of elicited understandings and misunderstandings. The categories, conceptual/procedural and common errors/overgeneralizations are used where appropriate and are written in italics. Figure 1.16 shows an example, Examining Student Work, from the Mathematics Assessment Probe, What's the Capacity?

***S**eeking links to cognitive research.* This section provides additional information about research that teachers can use for further study of the topic. Figure 1.17 shows an example from the Mathematics Assessment Probe, What's the Capacity?

Figure 1.16 Examining Student Work

Example: *What's the Capacity?*

Examining Student Work

The distracters may reveal *common misunderstandings* regarding geometric measurement such as a lack of *conceptual understanding* of how a change in the size of the base and height impacts the volume of a figure, or the *overgeneralization* about the size or shape of the cylinder.

- *The correct answer is A:* Figure A with an approximate volume of 93 cubic units has a greater volume than Figure B with an approximate volume of 88 cubic units). **(See Student Responses 1, 2, and 3)**

- *Distracter B:* Students answering B often are incorrectly applying the intuitive rule of More A - More B (Stavy and Tirosh, 2000): overgeneralizing that taller figures have greater volume. **(See Student Responses 4 and 5)**

- *Distracter C:* Students overgeneralize about the equivalent portion of the figure's net as shown in the diagram and so do not incorporate the size of the base. They often are incorrectly applying the intuitive rule of Same A - Same B (Stavy and Tirosh, 2000): since the initial 8.5 × 11 rectangle is the same, the volume must be equivalent. **(See Student Responses 6 and 7)**

Figure 1.17 Seeking Links to Cognitive Research

Example: *What's the Capacity?*

Seeking Links to Cognitive Research

Findings suggest that, as with area and length measure, students' models of spatial structure influence their conceptions of an object's volume measure. (NCTM, 2003, p. 186)

The intuitive rule, Same A–Same B, is activated in comparison tasks in which two objects are equal in a certain quantity and differ in another quantity. . . . When students start to conserve the area of two sheets of paper, they will also argue that the volumes of the two cylinders are equal. Our data show . . . a very small percentage of the students in the upper grades [6–9] correctly argued about the cylinder with the greater volume. "There is no evidence of a decrease with age in students' incorrect judgments about conservation of volume." (Stavy & Tirosh, 2000, pp. 51–52)

Teaching implications. Being aware of student difficulties and their sources is important, but designing instruction is most important to help diminish those difficulties. Although some ideas are included, the authors strongly encourage educators to use the Curriculum Topic Study (CTS) process to search for additional teaching implications. Each set of Teachers' Notes includes the related CTS Guide for further study, additional references, and a "teacher sound bite" from a field-tester of the probe. Figure 1.18 shows an example from the Mathematics Assessment Probe, What's the Capacity?

Figure 1.18 Teaching Implications

Example: *What's the Capacity?*

In order to support a deeper understanding for students in regard to geometric measurement, the following are ideas and questions to consider in conjunction with the research.

Focus Through Instruction

- Students should be provided opportunities to experiment with volumes of objects using concrete objects.
- Students need experience composing and decomposing two- and three-dimensional shapes.
- Composing and decomposing shapes should be used as a method of finding volumes for various three-dimensional objects.
- Students should develop formulas through inquiry and investigation.
- Students with prior knowledge about the relationship between perimeter and area (see the "Are Area and Perimeter Related?" probe) are better able to understand the relationship of surface area and volume.

Questions to Consider . . . ***when working with students as they grapple with the ideas of surface area and volume***

- Do the students understand how the dimensions of the base and the height of an object impact its volume?
- Over time, are students able to use properties and formulas to justify their answers and check their assumptions?

Teacher Sound Bite

Students easily jumped to conclusions based on a visual assumption, and it was difficult to convince them otherwise. Prior to using this probe, I was not explicit in helping students [connect their] understanding that two or more figures can have the same perimeter but different areas to new ideas concerning surface area and volume.

Additional References for Research and Teaching Implications:

NCTM (2000), *Principles and Standards for School Mathematics,.* pp. 243–245.

NCTM (2003), *Research Companion to Principles and Standards for School Mathematics,* p. 186.

Stepans et al. (2005), *Teaching for K–12 Mathematical Understanding Using the Conceptual Change Model,* pp. 103+.

Van de Walle (2007), *Elementary and Middle School Mathematics,* pp. 387–388.

Curriculum Topic Study

What's the Capacity?
Related CTS Guide:
Volume

Following the Teachers' Notes and sample student responses, adaptations, or variations of the Mathematics Assessment Probe are provided for some of the probes. Variations of the probe provide a different structure (selected response, multiple selections, opposing views, examples/non-examples, justified list, and strategy harvest) for the question within the same grade span. An adaptation to the probe is similar in content to the original, but the level of mathematics changes for use with a different grade span. In addition to the Teachers' Notes, a Note Template is included in Resource A. The Note Template provides a structured approach to working through a probe quest. The components of the template are described in Figure 1.19.

Figure 1.19 Note Template

Question to Answer

Uncover Understandings Using the Following Mathematics Assessment Probe

Adaptations made to the probe:

Examine student thinking:

Seek Additional Research Findings

Source:

Findings:

Source:

Findings:

Source:

Findings:

Teaching implications

Source:

Findings:

Source:

Findings:

Source:

Findings:

Summary of instructional implications/Plan of Action:

Results of Instruction:

WHAT MATHEMATICS ASSESSMENT PROBES ARE INCLUDED IN THE BOOK?

Many of the samples included in this book fall under numeric operations, symbolic representation, and geometric measurement because the cognitive research is abundant in these areas. The book also includes multiple examples for the following additional content standards: Numbers and Operations, Algebra, Data Analysis, Probability, Geometry, and Measurement. Figure 1.20 provides an "at a glance" look of the grade span and content of the probes included in Chapters 3 through 5. Grade-span bars are provided to indicate the developmentally appropriate level of mathematics as aligned to NCTM Standards as well as the cognitive research.

An important note to high school teachers: Many of the mathematics expectations in the Grades 6–8 span of the NCTM Standards are being introduced to some students for the first time in high school mathematics courses.

Figure 1.20 Grade Span and Content of the Probes

Grade Span Bar Key

	Target for Instruction Depending on Local Standards
	Prerequisite Concept/Field Testing Indicates Student Difficulty

Number and Operation Probes		**Grade Span Bars**	
Question	**Probe**	**Grade 6–8**	**Grades 9–12**
Can students choose all correct values of various digits of a given decimal?	What is the Value of the Digit? p. 33		
Do students correctly choose the various meanings of a/b?	What Is the Meaning of 2/3? p. 39		
Are students able to choose equivalent forms of a fraction?	Is It Equivalent? p. 45		
Can students use estimation to choose the closest benchmark to an addition problem involving fractions?	What's Your Estimate? p. 51		
Do students understand there are multiple methods of estimating the sum of three 3-digit numbers?	Is It an Estimate? p. 56		
Do students use the "canceling of zeros" shortcut appropriately?	Is It Simplified? p. 61		
Are students able to locate 1 million on a number line labeled from 0 to 1 billion?	Where Is One Million? p. 66		
Do students understand how various integer exponents affect the value of the numerical expression?	How Low Can You Go? p. 71		
When adding, can students apply and understand a variety of different strategies?	What's Your Addition Strategy? p. 76		
When subtracting, can students apply and understand a variety of different strategies?	What's Your Subtraction Strategy? p. 84		
When multiplying, can students apply and understand a variety of different strategies?	What's Your Multiplication Strategy? p. 92		
When dividing, can students apply and understand a variety of different strategies?	What's Your Division Strategy? p. 99		

Measurement, Geometry, and Data Probes			
Question	**Probe**	**Grade 6–8**	**Grades 9–12**
Are students able to choose the correct measure of a line given a change in the interval?	What's the Measure? p. 107		
Do students understand that figures can have the same perimeter but different areas?	Are Area and Perimeter Related? p. 112		
Do students recognize how a change in the dimensions affects the area of a figure?	What's the Area? p. 117		
Do students recognize how a change in the dimensions affects the volume of a figure?	What's the Capacity? p. 122		
Do students recognize dilations (reduction or contraction and enlargement or magnification) as types of transformation?	Is It Transformed? p. 126		
Are students able to identify needed information in determining whether two figures are similar?	Are They Similar? p. 131		
Do students understand mean and how it is affected by changes to a data set?	What Do You *Mean?* p. 136		
Are students able to move beyond point-by-point graph interpretation?	Name of the Graph? p. 145		
Can students identify correct graphical construction and accurate use of interval scale?	Graph Construction p. 151		

Algebra Probes			
Question	**Probe**	**Grade 6–8**	**Grades 9–12**
Are students able to identify equivalent expressions in the form of familiar formulas?	Equal or Not Equal? p. 158		
Do students misuse "key words" when writing expressions?	Is It the Same as a + b? p. 165		
Do students correctly apply knowledge of equality and relationships among quantities?	M & N's? p. 170		
Do students understand how to evaluate an expression of the form "ax"?	What's the Substitute? p. 175		
Do students understand the operation implied by concatenation of literal symbols and numbers?	Is It True? p. 180		
Do students understand appropriate methods and notations when solving for an unknown?	Solving Equations p. 187		
Are students able to identify various representations of an inequality?	Correct Representation of the Inequality? p. 194		
Are students able to identify when a literal symbol is being used as a variable?	Is It a Variable? p. 200		
Do students correctly use the distributive law when multiplying algebraic binomials?	Binomial Expansion? p. 207		
Do students recognize the characteristics of the graph of a quadratic function?	Is It Quadratic? p. 213		